Improvement of the Multifractal Method for Detection of Early Reflections

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Abstract: This paper presents the research that was aimed at examining the possibilities for improving the multifractal method for detecting early reflections in the room impulse response. Multifractal method for the detection of early reflections uses distribution of Hölder’s exponent calculated for the acoustic impulse response. Modifications of algorithm proposed in this paper perform filtering of the distribution of Hölder’s exponent to improve the detection of significant early reflections. The results obtained in this way provide guidance how to improve detection of early reflections and to make it more precise.

Keywords: Room acoustics, Impulse response, Multifractals, Early reflections, Self-similarity.

1 Introduction

Room impulse response is the fundamental source of information for the analysis of the room acoustic properties. The appearance of the room impulse response is predominantly determined by the geometry of the room and its acoustic treatment.

The initial part of the impulse response is an area where individual reflections can be observed. These are early reflections that can be easily recognized by their intensity and lower density because they are predominantly reflections that bounced just once or twice from the walls or ceiling of the room. Early reflections provide information about the room acoustic characteristics and have significant impact on our experience of sound in the room. This impact is perceived as increased loudness, changes in intelligibility of speech, music clarity and the impression of the auditory source width.

In literature [1, 2, 3] it was noted that the position of the reflection in the impulse response is associated with a particular perception of sound in the room. Reflections arriving within the first few milliseconds immediately after the direct sound are responsible for the perception of the arrival direction of the sound, i.e. the position of the sound source. Reflections located in the time interval between 30 ms and 50 ms are responsible for...
identifying tone of the sound, while reflections outside of 60 ms in the impulse response contribute to formation of the impression of ambience and echo [2]. The order of early reflections in impulse response is responsible for the phenomena in acoustics such as sound masking. One strong reflection following the direct sound may extend the effect of masking in time [4]. When observing individual reflection following the direct sound, its perception depends primarily on the level of the direct sound, the direction from which the observed reflections is coming from, the delay of the observed reflections in comparison to the direct sound and the characteristics of the direct sound. It has been determined that it is easier to perceive reflection if the directions from which the observed reflection and direct sound arrive do not coincide [3]. Also, if the delay of the observed reflection relative to the direct sound is larger then the perception of that reflection is easier. On the other hand, if early reflections are coherent they may pose a major problem due to their correlation with the direct sound. If coherent reflections are not attenuated enough they can significantly affect the perception of sound in the room. Due to superposition of direct sound and coherent reflections comb-filter occurs that can introduce sound coloration in the perceived sound through spectral changes, i.e. timbre that the listener perceive.

Considering the above mentioned facts it can be concluded that early reflections have a very large impact on the experience of sound in the room, hence their detection is an important topic in acoustics. Therefore the idea to develop an algorithm to automate the detection of early reflections in the impulse response was introduced.

This paper uses multifractals as a tool for detecting early reflections in the room impulse response. The work is continuation of research conducted by the authors on the application of multifractal theory in the analysis of the room impulse response. This paper presents improvements of the suggested method for early reflections detection with the goal to increase the accuracy in determining the exact location of the early reflections. Also the aim was to isolate only those reflections that have significant impact on the room acoustics. The paper is organized as follows: the second chapter provides an overview of previous methods for the detection of early reflections including multifractal method. Chapters III and IV give suggestions for improvements and present results of the proposed improvements.

2 Detection of Early Reflections

2.1 Overview of the methods for detection of early reflections

The problem of early reflections detection has already been studied by other authors [5–11]. These works are roughly distinguished by the fact that they are dealing with localization of early reflections in time (the location in
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impulse response signal) or in space (direction of arrival). Previous studies on
the subject of early reflections detection were often based on the usage of
correlation between the direct sound and early reflections. The method proposed
in literature [5] reliably detects one to five early reflections by using the
adaptive thresholds for the detection of reflections’ arrival time. Using the direct
component of the impulse response as a basis for verification in the matching
pursuit algorithm is proposed in [6]. However, it is not tested to what extent the
detected reflections are the early reflections of significant importance in the
impulse response. To solve the problem of locating early reflections in time
cross-wavelet transform was also used [7]. For artificially generated impulse
responses, this method gives quite accurate results discovering most of the early
reflections. Continuous cross-wavelet transformation and then a method for
segmentation are applied for the early reflections detection in [7, 8]. Loutridis
[9] demonstrated how the continuous wavelet transform can be used in the
analysis of the room impulse response, while the wavelet decomposition is
applied in [10]. Another possible option for the detection of reflections in
impulse response is the analysis of energy with time windows of short duration
[11]. Taking into account the fact that average energy flow in a diffuse field
equals zero, detection would be based on the calculation of local energy for the
parts of the impulse response and its comparison with the energies calculated in
previous and next iteration.

2.2 Detection of early reflections using multifractals

The latest in the series of methods for the early reflections detection uses
achievements and concepts offered by multifractals [14]. Fractal geometry is
based on the idea that seemingly complex forms in nature show a fundamental
property known as self-similarity [12]. In fact, no matter how complex is the
shape and/or dynamic behaviour of the system, if one looks carefully and
imaginatively enough can find forms on one scale that resemble to those on
other scales of magnification. Examples of such forms in the nature are
numerous: the seashore, tree top, the structure of the nervous system, the
decimal number system... Characteristic for these examples is that their
structures are similar at different scales of magnification. In the case of artificially
generated fractals, this dimension is a single value.

Natural objects and phenomena do not show strict fractal properties, even
when they are self-similar. Unlike the artificial fractals they can have just
statistical self-similarity. Natural fractals have a different fractal dimensions at
different scales of magnification, i.e. their self-similarity cannot be described by
a single value, their fractal dimension changes with magnification. If a
phenomenon expresses properties of self-similarity, which are not the same in
different scales of magnification but they are similar, then we are talking about
multifractals [13]. Characteristic of multifractals (MF) is that they cannot be characterized by a single value (fractal dimension), but a whole range of values is needed for their description, the so called multifractal spectrum (or singularity spectrum).

For quantitative description of multifractal properties in this study Hölder's exponent is used. This non-integer parameter labelled $\alpha$ describes local regularity of signal. Its distribution is the MF spectrum. The widespread method of calculating that parameter is a box-counting method (due to its simplicity and fast computing procedure). The procedure starts with dividing the object, i.e. impulse response signal $S$ into non-overlapping windows of length $\varepsilon$. These windows form subsets of data $S_i$, such that $S = \bigcup S_i$. Coarse Hölder's exponent $\alpha_i$ of set $S_i$ is then calculated using the formula (1):

$$\alpha_i = \frac{\ln(\mu(S_i))}{\ln(\varepsilon)}$$

Hölder's exponent $\alpha$ is obtained as the limit value of $\alpha_i$ as $\varepsilon$ approaches zero ($\varepsilon \to 0$) [12]:

$$\alpha = \lim_{\varepsilon \to 0} (\alpha_i)$$

where $\mu(S_i)$ is a measure that characterizes the corresponding subset $S_i$. As a measure $\mu(Si)$ for the given data set different values can be used, such as maximum, minimum, sum, deviations, etc. These measures are normalized to the sum of all of values for the measure that is observed. Distribution in this manner calculated values of the exponent $\alpha$ is the MF spectrum. It is defined by the function $f(\alpha)$, which displays distribution of the windows that have value of the Hölder’s exponent within a specific range $\alpha + d\alpha$. In natural fractals, function $f(\alpha)$ has the shape of a parabola, similar to the example shown in Fig. 1.

The values $\alpha$ and $f(\alpha)$, describe both local and global regularity of the signal under investigation. Small values of $\alpha$ indicate that the signal has weak local changes, while small values of $f(\alpha)$ indicate that the phenomenon which has that value of $\alpha$ is highly unlikely and vice versa. In MF analysis, parts of the impulse response with rapid changes in intensity represent the points where its regularity changes. These “anomalies” in the signal are considered as “defects” in its structure and deviation from the global regularity.

MF analysis can be applied for solving problems in signal processing as a robust method for describing and extracting features hidden in the huge amount of data. From the point of MF analysis, the appearance of early reflections can be interpreted as “points” in the impulse response where "defects" in its structure appear. In these locations, apart from sudden changes in signal
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intensity, its regularity also changes. According to the fact that high values of the exponent $\alpha$ indicate points where the signal changes locally a lot, the assumption was introduced that high values of Hölder's exponent $\alpha$ indicate the presence of strong ERs in the impulse response [14].

![Multifractal spectrum](image)

**Fig. 1** – Illustration of multifractal spectrum $f(\alpha)$ for impulse response recorded in a studio.

Detection of early reflections is performed with MF transformation that allows bidirectional mapping between input values (samples of impulse response) and corresponding values of $\alpha$ and $f(\alpha)$. By applying that procedure it is possible to extract signal components belonging to specified parts of the MF spectrum. Specifically, it is possible to indicate a certain range of values in the distribution of Hölder’s exponent ($\alpha_{\text{low}} - \alpha_{\text{high}}$) from the MF spectrum, and then determine which parts of the input signals are responsible for making that range of $\alpha$ values. In this analysis the classical procedure described by (1) was used for deriving $\alpha$. The procedure was modified to take into account the limitations that introduce the discrete representation of the impulse response signal. For calculating Hölder’s exponent as the measure, $\mu(S_j)$ was chosen the maximum for the given data set. This measure gives the highest values of $\alpha$ for early reflections.

Fig. 2 illustrates the proposed method for the early reflections detection on the example of the impulse response recorded in the model of reverberation room. The middle image shows the distribution of calculated Hölder's
exponents for the given impulse response (upper image). As already mentioned, the assumption is that the large values of exponents $\alpha$ indicate points in the impulse response where significant early reflections are located. Method for detection could then be seen as the process of extracting the largest values of exponents, or the range ($\alpha_{\text{lower}} - \alpha_{\text{max}}$) from the obtained spectrum.

Fig. 2 – The illustration of the ER detection procedure: a) top image - analyzed impulse response; b) middle – calculated distribution of Hölder’s exponent and preferred range, $\alpha_{\text{low}} - \alpha_{\text{max}}$; c) bottom image – indication of places where ERs are detected – extracted ER for range $0.191 < \alpha < 0.907$ and original sequence.
In the example from Fig. 2 the lower limit $\alpha_{\text{low}}$ is chosen in such way as to extract 10 largest values of the exponent $\alpha$. From the exponents $\alpha$ separated in this way, using the inverse procedure reflections which generate the occurrence of these extracted values $\alpha$ are identified in the impulse response. Reflections detected in this way are shown in the lower figure (marked with arrows). Number of detected reflections directly depends on the selected range of exponent $\alpha$ values. The obtained results show compatibility between the reflections detected using multifractals and those observed with the naked eye. The results show that it is possible to detect most of the early reflections, but also there is a need to improve the accuracy of detection [14].

3 Possibilities for Improvement of Multifractal Method for the Detection of Early Reflections

Application of multifractals for detecting early reflections provides a different view on the impulse response structure. From the perspective of multifractals early reflections represent structures in the impulse response signal that are connected with the common parameter, the value range of the exponents $\alpha$. Therefore, proper selection of exponents $\alpha$ is the basic problem in the correct detection of early reflections. Efforts to improve the detection method are aimed at filtering values of exponent $\alpha$ to find those that indicate location of the early reflections. Efforts were also directed towards automating the method of determining the range of values of the exponent $\alpha$. Considering that besides the preferred reflection by this method are also detected other reflections that are not of importance for the acoustic analysis of the room, it is necessary to make changes in the algorithm to improve detection. Optimal detection would imply the detection of all significant early reflections in the impulse response, with minimal detection of reflections that are not relevant. Important question that needs to be answered at this moment is: which reflection should be detected, and what we mean by the term significant early reflections?

The term significant early reflections would include all reflections that may have significant impact on the perceived sound in the room.

Certainly the most important criterion to consider some reflection a significant early reflection besides location in the impulse response is its intensity. However, one should take into consideration other features that make one reflection a significant (early) reflection. These features are: relationship with the adjacent reflections (relative position and intensity in comparison to other early reflections) and correlation with the direct sound. The advantage of using multifractals for detection of early reflection is manifested in the fact that they use properties of self-similarity, so they are able to detect reflections that are correlated with the direct sound.
In order to improve results of early reflections detection, authors have attempted to increase the accuracy of detection by additional filtering the calculated values of Hölder’s exponent. Modifications were made to achieve the following objectives: reducing the false positive detections by discarding the small values of the exponents and reducing the impact of adjacent reflections through the rejection of reflections that are masked. Modification of multifractal method for detection went in several directions.

The first modification attempted to extract the highest values of exponent $\alpha$ within window of a specified width which shifts without overlaps over calculated values. Each time the window shifts maximum value $\alpha_{\text{max}}$ within the window is recorded. If the value is significantly higher than its neighbours within the window then that value $\alpha_{\text{max}}$ is extracted from the window. If that value does not exceed the specified criteria, then the value of $\alpha_{\text{max}}$ within the window remains unchanged. The proposed algorithm named Modification1 is illustrated in Fig. 3. Result is a modified distribution of exponent $\alpha$ values over which is then once again selected range of the largest values.

![Fig. 3 – Illustration of algorithm Modification1.](image-url)
The second modification also uses window of fixed width (equal to a specified number of samples). This window shifts over the distribution of Hölder’s exponent without overlapping; in every shift maximum value $\alpha_{\text{max}}$ within the window is found. This value is then compared with the effective value of the exponent $\alpha_{\text{eff}}$ calculated within the window. The difference between maximum and effective value ($\alpha_{\text{max}} - \alpha_{\text{eff}}$) in each window is recorded at the location of $\alpha_{\text{max}}$ in the window. Other values of exponent $\alpha$ within the window are set to zero. The window is then shifted for the number of samples that matches the width of the window and the above procedure repeats. The proposed algorithm for extracting exponents $\alpha$ named Modification2 is illustrated in Fig. 4. The result is new modified distribution of values over which once again largest values of exponent are extracted, and detection of early reflections is then performed.

![Fig. 4 – Illustration of algorithm Modification2.](image)

The third approach included the analysis of correlation coefficients that are calculated between the direct sound and reflections in impulse response. Using this coefficients specified range of Holder’s exponents ($\alpha_d - \alpha_g$) was filtered. The idea in this case was to give advantage to the reflections which are highly correlated with the direct sound. In the obtained distribution of correlation coefficients only those correlation coefficients that have high values (marked with the green arrows in the Fig. 5) were retained.
Using these correlation coefficients, values of the Hölder’s exponents are modified in terms that only the values of exponent $\alpha$ which correspond to the location of retained coefficients of correlation were kept. The method named Modification3 is illustrated in Fig. 5.

4 Comparison of the Proposed Methods for Improvement

Comparing the effectiveness of the proposed methods for detection of early reflections is done using the control reflections that are inserted into specific parts of the impulse response. Assessing the effectiveness of the proposed algorithms requires knowledge about the exact locations of all early reflections in the impulse response. Due to unreliability of their naked eye recognition, especially their coherence with the direct sound, the evaluation of the proposed detection method was carried out using control reflections (CR) artificially inserted in impulse response signal. The control reflections were generated from real impulse responses as the attenuated replicas of the first impulse (direct sound). Such control reflections were inserted into the impulse response at pre-defined locations. The basic idea for assessing proposed detection methods was to investigate whether and under what conditions it is possible to detect control reflections inserted in the impulse response. Inserted CRs were successively decreased in intensity to the level at which they cease to be detected. That value of the CR level is registered as $L_{CR}$. Also the level of local maximum $L_{loc}$ in 5 ms width interval of impulse response signal around each control reflection is measured.
The experiment was organized using the following types of impulse response signals:

1. Two impulse responses recorded in two opera halls (IR1 and IR2), and
2. Two impulse responses recorded in the laboratory using a scaled reverberation chamber model (IR3 and IR4). The reverberation chamber is scaled 1:10, the recorded impulse responses were later transformed in full scale.

In the assessment of CR detection effectiveness the following parameters were observed: The maximum difference between the direct sound levels $L_{FI}$ and $L_{CR}$ ($L_{FI} - L_{CR}$) – shows a relative decrease of the control reflection level in the impulse response in the moment when its detection ceases. The minimum difference between $L_{CR}$ and the local maximum level $L_{loc}$ ($L_{CR} - L_{loc}$) – shows how much the control reflection level deviates from the adjacent signal. The first parameter quantifies the strength (effectiveness) of the proposed method, while the second quantifies its sensitivity.

The Fig. 6 shows the comparison of the effectiveness assessment for the proposed algorithms on the example of control reflections detection. The diagrams show the limit at which the detection of CR inserted in the impulse response signal stops.

Obtained results display improvements achieved with the proposed algorithms for detection. The detection thresholds are shifted averagely by more than 5 dB. Using these modifications it is possible to detect low-intensity reflections that are correlated with the direct sound. Fig. 7 and Table 1 give comparison of the proposed algorithms on the example of real reflections detection in the impulse response.
Results show that the proposed algorithms for the improvement of the detection method are able to detect more early reflections. The results given in Table 1 assume that the reflections that were identified with the naked eye are representative for the comparison. Problem with the visual inspection is that it considers only the intensity of reflections, and something that is not so easy to see in the impulse response is the form of these reflections and their structure. This is exactly where the intervention from the multifractal standpoint is necessary to assist in proper detection of early reflections of interest.

### Table 1

Comparison of proposed algorithms for detection on the example of real reflections (x - detected, o - not detected).

<table>
<thead>
<tr>
<th>No.</th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
<th>R5</th>
<th>R6</th>
<th>R7</th>
<th>R8</th>
<th>No. of det. refl.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original code</td>
<td>x</td>
<td>o</td>
<td>x</td>
<td>o</td>
<td>x</td>
<td>o</td>
<td>x</td>
<td>o</td>
<td>4</td>
</tr>
<tr>
<td>Modification 1</td>
<td>x</td>
<td>o</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>o</td>
<td>x</td>
<td>x</td>
<td>6</td>
</tr>
<tr>
<td>Modification 2</td>
<td>x</td>
<td>o</td>
<td>x</td>
<td>o</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>6</td>
</tr>
<tr>
<td>Modification 3</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>o</td>
<td>o</td>
<td>x</td>
<td>6</td>
</tr>
</tbody>
</table>
5 Conclusion

Method for detection of early reflections based on the multifractal theory, i.e. on distribution of Hölder’s exponents, uses the local properties of the signal to locate early reflections. This method can be used as an additional tool for the evaluation of the room acoustic properties by automating the detection of early reflections. The results that were obtained using multifractals for detection of early reflections show justification for their use.

In order to make detection results better, improvements of multifractal method were proposed. The main directions in which the improvements were made, are reduction of false positive detection and emphasizing strong reflections. In order to increase detection of coherent reflections the correlation coefficient between the direct sound and reflections were used in combination with the Hölder’s exponents calculated for the impulse response signal.

The results of the proposed modifications show that to improve the precision it is necessary to take into account the additional factors in the selection of Hölder’s exponent which indicate early reflections. Factors that must be considered are: the relative position of reflections with respect to the adjacent reflections, as well as the subjective experience of sound in the room.

The results obtained using the modified detection algorithms with multifractals show that the proposed methods are able to detect control reflections that are attenuated up to 15 dB compared to the original. In case of real reflections detection proposed methods detect most of the wanted reflections. The assumption that early reflections are connected with the common parameter (the range of Hölder’s exponent values), although justified requires further improvement. Some potential options for improvement are presented in this paper. Inclusion of the achievements of psychoacoustics in defining the criteria for rejecting and detection of early reflections is a task for future research on the application of multifractals for the detection of early reflections.

6 References

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