Series PID Pitch Controller of Large Wind Turbines Generator

Aleksandar D. Micić, Miroslav R. Mataušek

Abstract: For this stable process with oscillatory dynamics, characterized with small damping ratio and dominant transport delay, design of the series PID pitch controller is based on the model obtained from the open-loop process step response, filtered with the second-order Butterworth filter \( F_{bw} \). Performance of the series PID pitch controller, with the filter \( F_{bw} \), is analyzed by simulations of the set-point and input/output disturbance responses, including simulations with a colored noise added to the control variable. Excellent performance/robustness tradeoff is obtained, compared to the recently proposed PI pitch controllers and to the modified internal model pitch controller, developed here, which has a natural mechanism to compensate effect of dominant transport delay.

Keywords: PID pitch control, Wind turbines generator, Oscillatory dynamics, Dominant dead-time, Modified internal model control.

1 Introduction

Recently, synthesis on PI-based pitch controller of large wind turbines generator is considered in [1, 2], where the process dynamics is defined by the following transfer function

\[
G_p(s) = G_{p0}(s)e^{-\tau s}, \\
G_{p0}(s) = \frac{a_2s^2 + a_1s + a_0}{s^4 + b_3s^3 + b_2s^2 + b_1s + b_0},
\]

(1)

with parameters in Table 1 [1], for two processes denoted as \( G_{p1}(s) \) and \( G_{p2}(s) \).

It is supposed in [1, 2] that hydraulic pitch actuator AC is applied, modeled as a dead-time \( \tau \). Thus, process is defined by \( AC = \exp(-\tau s) \) in series with \( G_{p0}(s) \), as in (1). Transfer function (1) defines a single-input single-output process, where input is the blade pitch angle and output defines tower fore-aft deflection. Processes \( G_{pi}(s), i = 1, 2 \), exhibit a strong resonant response even by the small amount of excitation which is naturally present in the wind [3].

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Table 1
Parameters of $G_{p1}(s)$ and $G_{p2}(s)$.

<table>
<thead>
<tr>
<th>Process</th>
<th>$a_2$</th>
<th>$a_1$</th>
<th>$a_0$</th>
<th>$b_3$</th>
<th>$b_2$</th>
<th>$b_1$</th>
<th>$b_0$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{p1}(s)$</td>
<td>-0.6219</td>
<td>-8.7165</td>
<td>-2911</td>
<td>5.018</td>
<td>691.3</td>
<td>1949</td>
<td>1.15 · 10^5</td>
<td>0.25</td>
</tr>
<tr>
<td>$G_{p2}(s)$</td>
<td>2.426</td>
<td>-4.6345</td>
<td>-147.3</td>
<td>4.857</td>
<td>126.2</td>
<td>266.4</td>
<td>3.66 · 10^3</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Design of pitch controllers based on the process dynamics characterization (1), as in [1, 2], is a complex control problem: to design a controller with a good performance/robustness tradeoff for the process with oscillatory dynamics, characterized with small damping ratio and dominant transport delay. Investigation presented in [4] is performed to demonstrate how to apply optimization to tune parallel PID controller for process (1) with strong resonant response.

In the present paper design of pitch controllers is based on the methods proposed for series PID controller in [5] and for Modified Internal Model Control (MIMC) in [6]. In Section 2, control relevant models $G_{pid}(s)$, and $G_{mimc2}(s)$, of processes $G_{p1}(s)$ and $G_{p2}(s)$, are determined by applying a simple procedure. In Section 3, design of PID and MIMC pitch controllers are performed based on models determined in Section 2. As demonstrated in Section 3, MIMC controller has a natural mechanism to compensate effect of dominant dead-time. This is the reason why the MIMC pitch controller, providing excellent performance/robustness tradeoff, is also used to demonstrate properties of the proposed series PID pitch controller. In Section 4, closed-loop simulation results are presented and compared with results obtained by PI controllers from [1, 2]. Simulations with a colored noise added to the control variable are also presented and used to simulate effect of the stochastic wind variation on the tower fore-aft deflection. Finally, simulations with strong rate constraints in actuator are used to demonstrate advantages of the series PID pitch controller and PI pitch controller proposed here.

2 Models Used in the Proposed Pitch Controller Design

Control relevant models $G_{mimc2}(s)$, used for the MIMC pitch controller design, are defined by

$$G_{mimc}(s) = G_{mimc0}(s)e^{-t_{mimc}s}, \quad G_{mimc0}(s) = K \frac{T_zs + 1}{T^2s + 2\xi Ts + 1}$$

and Table 2. Simple process dynamics characterization is used to obtain control relevant models (2) by applying fitting-by-eye technique to approximate open-loop step responses of stable processes $G_{p1}(s)$ and $G_{p2}(s)$.
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Table 2
Parameters of models (2) used in the MIMC pitch controller design.

<table>
<thead>
<tr>
<th>Process/Model</th>
<th>$K$</th>
<th>$T$</th>
<th>$T_z$</th>
<th>$\xi$</th>
<th>$\tau_{mimc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{p1}(s)/G_{mimc1}(s)$</td>
<td>-0.0253</td>
<td>0.05</td>
<td>0.10</td>
<td>0.018</td>
<td>0.42</td>
</tr>
<tr>
<td>$G_{p2}(s)/G_{mimc2}(s)$</td>
<td>-0.0402</td>
<td>0.14</td>
<td>0.42</td>
<td>0.060</td>
<td>0.51</td>
</tr>
</tbody>
</table>

Responses of model (2), compared to responses of process $G_p(s)$ in (1), are presented in Fig. 1. From Fig. 1, and Table 2, it follows that the oscillatory dynamics of processes $G_{p1}(s)$ and $G_{p2}(s)$ is characterized by small damping ratio $\xi$ and dominant dead-time $\tau_{mimc}$, larger than time constant $T$. The nonminimum-phase characteristics of $G_{p2}(s)$ response are included in the dead-time of model $G_{mimc2}(s)$, as demonstrated in Fig.1b. Similar values of the time constant $T=0.1398$ and damping ratio $\xi=0.0618$, as in the second row of Table 2, are obtained from the dominant complex-conjugate pole pair of $G_{p2}(s)$, at $s_1=-0.4416+7.1373i$ and $s_2=-0.4416-7.1373i$.

![Fig. 1](image1)

Fig. 1 – Unit step response of $G_{pj}(s)$, $j = 1, 2$, (dotted) and models $G_{mimc,j}(s)$ (solid): a) $G_{p1}(s)$ and $G_{mimc1}(s)$, b) $G_{p2}(s)$ and $G_{mimc2}(s)$.

Control relevant models $G_{pid}(s)$ are obtained by applying fitting-by-eye technique to approximate filtered open-loop step responses $G_p(s)F_{bw}(s)$ of stable processes $G_{p1}(s)$ and $G_{p2}(s)$, with oscillatory dynamics, as proposed in [5]. The second-order Butterworth filter $F_{bw}(s)$ is used here in the form

$$F_{bw}(s) = \frac{1}{T_{fb}^2 s^2 + \sqrt{2} T_{fb} s + 1}.$$  (3)

Models $G_{pid}(s)$ are defined by Table 3 and
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\[ G_{\text{pid}}(s) = G_{\text{pid}0}(s)e^{-\tau_{\text{pid}}}, \quad G_{\text{pid}0}(s) = K \left/ \left[ \left( T_1s + 1 \right) \left( T_2s + 1 \right) \right] \right., \quad T_1 = T_2. \]  

(4)

**Table 3**

Parameters of models (4) used in the series PID pitch controller design.

<table>
<thead>
<tr>
<th>Process/Model</th>
<th>( T_{fb} )</th>
<th>( K )</th>
<th>( T_1 )</th>
<th>( T_2 )</th>
<th>( \tau_{\text{pid}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G_{p1}(s)/G_{\text{pid}1}(s) )</td>
<td>1.00</td>
<td>-0.0253</td>
<td>0.72</td>
<td>0.72</td>
<td>0.5</td>
</tr>
<tr>
<td>( G_{p2}(s)/G_{\text{pid}2}(s) )</td>
<td>1.41</td>
<td>-0.0402</td>
<td>1.00</td>
<td>1.00</td>
<td>0.6</td>
</tr>
<tr>
<td>( G_{p2}(s)/G_{\text{pid}2,2}(s) )</td>
<td>2.00</td>
<td>-0.0402</td>
<td>1.50</td>
<td>1.50</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Process responses and filtered responses \( G_p(s)F_{bw}(s) \), with \( F_{bw}(s) \) from (3) and Table 3, are compared in Figs. 2 and 3 to responses of models \( G_{\text{pid}}(s) \) from (4) and Table 3. Correct values of parameters for model \( G_{\text{pid}1}(s) \), denoted in [4] as \( G_{\text{SO1}}(s) \), are presented in Table 3.

![Fig. 2](image1.png)

(a) Determination of model used to design series PID1 controller: a) unit step response of \( G_{p1}(s) \) (dotted) and filtered response \( G_{p1}(s)F_{bw}(s) \) with \( T_{fb} = 1 \) (solid), b) filtered response (solid) and model response \( G_{\text{pid}1}(s) \) (dashed).

![Fig. 3](image2.png)

(b) Determination of models used to design series PID2 controllers: a) unit step response of \( G_{p2}(s) \) (dotted) and filtered responses \( G_{p2}(s)F_{bw}(s) \) with \( T_{fb} = 1.41 \) (solid-magenta) and \( T_{fb} = 2 \) (solid-red), b) filtered responses and model responses \( G_{\text{pid}2,1}(s) \) (dashed-magenta) and \( G_{\text{pid}2,2}(s) \) (dashed-red). (Colors can be seen in electronic version),
3 Control System Design and Tuning

Structure of the MIMC pitch controller is presented in Fig. 4. Transfer function \( G_{\text{mimc}0}^{-1}(z) = K^{-1}W_s(z)W_p(z) \) is a discrete inverse of the model \( G_{\text{mimc}0}(s) \)[6], defined by parameters in Table 2 and adopted sampling period \( \Delta t \). By applying a discretization procedure [6], from (2) one obtains

\[
W_p(z) = \frac{z^2 + a_{p1}z + a_{p2}}{1 + a_{p1} + a_{p2}}z, \quad a_{p2} = \exp(-2\xi\Delta t / T),
\]

while

\[
a_{p1} = -2\exp(-\xi\Delta t / T)\cos\left(\frac{\Delta t}{T}\sqrt{1-\xi^2}\right),
\]

Transfer function \( G_L(z) = z^{-(L+1)} \), where \( L \approx \tau_{\text{mimc}} / \Delta t \). Filter \( F(z) \) in Fig. 4, for \( F(z) \equiv 1 \) defines the desired closed-loop system time constant \( T_{CL} \) [6]. In the present paper \( F(z) \) is designed as a discrete equivalent of the second-order low-pass filter \( 1/(T_c s + 1)^2 \)

\[
F(z) = \left(\frac{(1-\alpha)z}{z-\alpha}\right)^2, \quad \alpha = \exp(-\Delta t / T_c), \quad T_c = 0.4.
\]

Tuning parameters \( T_c \), and \( \alpha \) in (7), are defined by the desired value of the time constant \( T_{CL} \), for the second-order filter equal to \( T_{CL} \approx 2T_c \). The adopted value of \( T_c = 0.4 \text{ s} \) corresponds to the closed-loop system time constant obtained with PI controllers from [1, 2]. Since \( T_{CL} \approx 0.8 \text{ s} \), the sampling period \( \Delta t = 0.05 \text{ s} \) is used to define filter \( F(z) \) in (7), with \( \alpha = 0.8825 \), and to design MIMC\(_j\), \( j = 1, 2 \) controllers.

Fig. 4 – MIMC pitch control system structure with set-point prefilter \( F_r(z) \), Zero-Order Hold (ZOH) and actuator AC. Input and output disturbances are denoted by \( d \) and \( n \).
Thus, for $G_{p1}(s)$, from $G_{mimc1}(s)$ and $\Delta t = 0.05$ s, one obtains in the MIMC$_1$ controller $G_{mimc0}^{-1}(z) = K^{-1}W_2(z)W_p(z)$,

$$G_{mimc0}^{-1}(z) = (-0.0253)^{-1} \frac{(1-0.606)z - 1.0616z + 0.9646}{z-0.606 (1-1.0616 + 0.9646)z^2},$$

(8)

while for $G_{p2}(s)$, from $G_{mimc2}(s)$ and $\Delta t = 0.05$ s, one obtains in the MIMC$_2$ controller

$$G_{mimc0}^{-1}(z) = (-0.0402)^{-1} \frac{(1-0.8878)z - 1.8345z + 0.9580}{z-0.8878 (1-1.8345 + 0.9580)z^2},$$

(10)

$$G_L(z) = z^{-9},$$

(9)

(11)

MIMC controllers are defined by Fig. 4 where $F(z)$, $G_{mimc0}^{-1}(z)$ and $G_L(z) = z^{-(L+1)}$ are defined by (7) – (11). The same filter $F(z)$ in (7) with $\alpha=0.8825$ is used in both MIMC$_j$, $j=1,2$ controllers, implemented with sampling period $\Delta t = 0.05$ s.

Anti-Reset Windup (ARW) implementation from [5], presented in Fig. 5, is used for the PID pitch controller with parameters $K_p$, $T_i$, $T_d$, $T_{fb}$ defined in Table 4. To obtain satisfactory set-point following response this implementation requires a set-point prefilter $F_r(s) = F_{bw}(s)$.

Fig. 5 – Anti-reset windup implementation of the series PID pitch controller.

For $AC = \exp(-\tau s)$ one obtains linear case, without amplitude or rate constraints in actuator AC. Set-point prefilter $F_r(s) = F_{bw}(s)$.

Parameters of the series PID pitch controller in Fig. 5 are given in Table 4. They are obtained by applying models $G_{p}(s)$ defined by (4), Table 3 and Simple Control (SIMC) tuning rules from [7] used to determine proportional gain, integral time $T_i$ and derivative time $T_d$ as proposed in [7]. According to Table 3, it is used $T_1 = T_2$ in (12). Besides the Butterworth filter $F_{bw}$, series PID pitch controller includes a low-pass filter $1/(T_j s + 1)$ in the term defining derivative action $(T_d s + 1)/(T_j s + 1)$, where $T_j = T_d / 10$. 

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\[ K_p = \frac{T_i}{2K \tau_{pid}}, \quad T_2 \leq T_i, \quad T_i = \min\{T_1, 8\tau_{pid}\}, \quad T_d = T_2. \tag{12} \]

\begin{table}[h]
\centering
\caption{Parameters of series PID pitch controllers.}
\begin{tabular}{|c|c|c|c|c|}
\hline
Controller/Process & \(K_p\) & \(T_i\) & \(T_d\) & \(T_{fb}\) \\
\hline
PID\_1/\(G_{p1}(s)\) & \(-28.46\) & 0.72 & 0.72 & 1.00 \\
PID\_2,1/\(G_{p2}(s)\) & \(-20.73\) & 1.00 & 1.00 & 1.41 \\
PID\_2,2/\(G_{p2}(s)\) & \(-31.09\) & 1.50 & 1.50 & 2.00 \\
\hline
\end{tabular}
\end{table}

According to results in Table 4, series PID pitch controllers can be implemented also as digital controllers by using sampling period \(\Delta t = 0.05\) s. However, they are implemented as continuous controllers, defined by Fig. 5, and parameters in Table 4. Continuous PI pitch controllers from [1, 2] are implemented as continuous controllers, in a standard way, with proportional and integral gains \(K_p\) and \(K_i\) defined by Table 5.

\begin{table}[h]
\centering
\caption{Parameters of PI pitch controllers from [1, 2].}
\begin{tabular}{|c|c|c|}
\hline
Controller/process & \(K_p\) & \(K_i\) \\
\hline
PI\_1/\(G_{p1}(s)\) & 1.0242 & \(-20.5914\) \\
PI\_2/\(G_{p2}(s)\) & 1.0000 & \(-20.0000\) \\
\hline
\end{tabular}
\end{table}

4 Results of Closed-Loop Simulation

As in [1, 2] in linear case, actuator is modeled as the dead-time, \(AC = \exp(-0.25s)\). Anti-Reset Windup (ARW) implementation from [5] presented in Fig. 5, is used for the PID pitch controller, with parameters \(K_p, T_i, T_d, T_{fb}\) in Table 4.

Closed-loop system responses obtained by the PID pitch controllers are compared in Figs. 6 – 9 with results obtained by the MIMC pitch controller with \(F_r(z) \equiv 1\), and obtained by PI pitch controllers from [1, 2] defined by Table 5. Parameters of the PI\_1 controller are obtained in [2] by unconstrained minimization of IAE\_r, the integrated absolute error following the unit step set-point response. Parameters of the PI\_2 controller are taken from the stabilizing region in the \(K_i - K_p\) plane [1, Fig. 2, for \(K_i < 0\)], as a good compromise between performance and robustness.
The desired closed-loop system time constant equal to $2T_C = 0.8$ s, is satisfied almost exactly by applying MIMC controllers. For better performance, demonstrated in Fig. 6a, and similar performance in Fig. 7a, better robustness is obtained by the PID and MIMC controllers, as demonstrated by the robustness indices in Table 6, where performance/robustness tradeoff obtained by PI, PID and MIMC controllers is presented.

![Fig. 6 – Closed-loop system responses of nominal process $G_{p1}(s)$, with controllers: MIMC1 (solid), PID1 (dashed) and PI1 (dotted). Input and output disturbances: $D(s) = 30\exp(-20s)/((2s+1)s)$ and $N(s) = 0.3\exp(-40s)/s.$](image)

![Fig. 7 – Closed-loop system responses of nominal process $G_{p2}(s)$ with controllers: MIMC2 (dash-dot), PID2,1 (solid), PID2,2 (dashed) and PI2 (dotted). Input and output disturbances: $D(s) = 15\exp(-15s)/((2s+1)s)$ and $N(s) = 0.15\exp(-30s)/s.$](image)

Performance index $IAE_n$, in Table 6, is the integrated absolute error following the unit step output disturbance $N(s) = 1/s$, obtained in simulation with the input disturbance $D(s) = 0$ and set-point $R(s) = 0$.

Variances in Table 6, are calculated as
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\[ \sigma_y^2 = \int_0^{T_{\text{sim}}} y(t)^2 \, dt / T_{\text{sim}}, \]  

where \( T_{\text{sim}} \) is the simulation time interval \( T_{\text{sim}} = 60 \text{ s} \) and \( y(t) \) is the controlled variable response. Simulation is performed with the input disturbance \( D(s) = 0 \), set-point \( R(s) = 0 \) and output disturbance \( n(t) \) defined by a Band-Limited White Noise (BLWN) \( n(t) = n_w(t) \). It is obtained from a BLWN generator with power \( b_w = 0.0005 \) and sample time \( T_s = \Delta t = 0.05 \text{s} \). The variance of this BLWN \( n_w(t) \) is theoretically equal to \( b_w/T_s = 0.01 \) [8, Appendix A]. Thus, as demonstrated in Table 6, all controllers guarantee low value of variance \( \sigma_y^2 \).

**Table 6**

Performance/robustness tradeoff obtained with PI, MIMC and PID controllers.

<table>
<thead>
<tr>
<th>Controller/Process</th>
<th>IAE</th>
<th>( \sigma_y^2 )</th>
<th>( M_S )</th>
<th>( M_T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI(<em>1/)( G</em>{p1}(s) )</td>
<td>1.93</td>
<td>0.01139</td>
<td>4.01</td>
<td>3.06</td>
</tr>
<tr>
<td>MIMC(<em>1/)( G</em>{p1}(s) )</td>
<td>1.17</td>
<td>0.01037</td>
<td>1.38</td>
<td>1.00</td>
</tr>
<tr>
<td>PID(<em>1/)( G</em>{p1}(s) )</td>
<td>1.42</td>
<td>0.01056</td>
<td>1.62</td>
<td>1.20</td>
</tr>
<tr>
<td>PI(<em>2/)( G</em>{p2}(s) )</td>
<td>1.26</td>
<td>0.01147</td>
<td>2.17</td>
<td>1.40</td>
</tr>
<tr>
<td>MIMC(<em>2/)( G</em>{p2}(s) )</td>
<td>1.49</td>
<td>0.01043</td>
<td>1.54</td>
<td>1.01</td>
</tr>
<tr>
<td>PID(<em>2,1/)( G</em>{p2}(s) )</td>
<td>1.81</td>
<td>0.01060</td>
<td>1.62</td>
<td>1.25</td>
</tr>
<tr>
<td>PID(<em>{2,2}/)( G</em>{p2}(s) )</td>
<td>1.96</td>
<td>0.01055</td>
<td>1.54</td>
<td>1.28</td>
</tr>
</tbody>
</table>

**Fig. 8** – Closed-loop system set-point responses of perturbed process \( G_{p2}(s) \), with controllers: PID\(_{2,2} \) (dashed), MIMC\(_2 \) (solid) and PI\(_2 \) (dotted). In a) perturbed gain, \( G_{p2,\text{pert}}(s) = 2.04 G_{p2}(s) \), b) perturbed dead-time, greater 44%, \( G_{p2,\text{pert}}(s) = G_{p2}(s) \exp(-0.11 \text{s}) \).
It is important that performance/robustness tradeoff obtained by the series PID pitch controller depends weakly on the choice of the time constant $T_{fb}$. This is demonstrated in Figs. 7a and 7b and confirmed in Table 6, for two values $T_{fb}=1.41$ and $T_{fb}=2$, resulting into practically equal performance/robustness tradeoff for evidently different model responses in Fig. 3b.

Excellent responses are obtained for the perturbed process $G_{p2}(s)$, with the proposed PID$_{2,2}$ pitch controller compared in Fig. 8 to MIMC$_2$ dead-time compensating pitch controller and PI$_2$ pitch controller. High values of robustness indices $M_S$ and $M_T$ in Table 6, obtained by the PI$_1$ controller, are consequence of unconstrained optimization applied in [2].

Robustness indices, maximum sensitivity $M_S$ and maximum complementary sensitivity $M_T$, are defined by the sensitivity function $S(s) = 1/(1+L(s))$ and complementary sensitivity $T(s) = 1-S(s)$ as

$$M_S = \max_{\omega} |S(i\omega)|, \quad M_T = \max_{\omega} |T(i\omega)|. \tag{14}$$

Loop transfer function is defined by $L(s) = G_{p0}(s)C(s)$. From Fig. 4 and $AC= \exp(-\tau s)$, one obtains that $C(s)$, in $L(s) = G_{p0}(s)C(s)$, used to obtain robustness indices $M_S$ and $M_T$ for MIMC controllers, is given by

$$C(s) = e^{-\tau s} G_{mimc0}^{-1}(z) F(z) / (1 - F(z)G_L(z))|_{z=\exp(s\Delta t)}. \tag{15}$$

For the series PID controllers, from Fig. 5 and $AC= \exp(-\tau s)$, one obtains $C(s)$ given by

$$C(s) = K_p \frac{T_i s + 1}{T_i s} e^{-\tau s} \frac{T_d s + 1}{0.1T_d s + 1} F_{bw}(s), \tag{16}$$

in $L(s) = G_{p0}(s)C(s)$. For PI controllers, PI$_1$ from [2] and PI$_2$ from [1], one obtains

$$C(s) = \left( K_p + \frac{K_i}{s} \right) e^{-\tau s}. \tag{17}$$

Finally, to illustrate influence of wind speed variation on the tower fore-aft deflection, simulation is performed with the set-point $R(s)=0$, output disturbance $N(s)=0$ and with noise $n_{wind}(t)$ acting as an unknown input disturbance $d(t) = n_{wind}(t)$. Stochastic variation of the wind speed $n_{wind}(t)$ is obtained from a BLWN generator with power $b_w = 0.0001$ and sample time $T_s = 0.01$ s, passed through the low-pass filter $F(s) = 1/(5s + 1)$. Results of this simulation are summarized in Fig. 9 and Table 7. Variances presented in Table 7 are calculated as in (13) for the simulation time interval $T_{sim} = 60$ s.
Random variation of the simulated wind speed acts on the blade pitch angle as presented in Fig. 9a. As a result the tower fore-aft deflection is obtained as in Fig. 9b. Smaller variation of the tower fore-aft deflection is obtained by the MIMC and PID controllers, with significantly smaller control signal activity, moves up and down of the manipulated variable \( w(t) \), demonstrated in Fig. 9a.

![Fig. 9](image)

(a) (b)

**Fig. 9** – A part of closed-loop system responses of nominal process \( G_{p1}(s) \) with controllers: PI\(_1\) (dotted), PID\(_1\) (solid) and MIMC\(_1\) (dashed). Only input disturbance is active, simulating stochastic variation of the wind speed.

<table>
<thead>
<tr>
<th>Controller/Process</th>
<th>( \sigma^2_{y} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI(<em>1)/( G</em>{p1}(s) )</td>
<td>22.23e-09</td>
</tr>
<tr>
<td>PID(<em>1)/( G</em>{p1}(s) )</td>
<td>9.69e-09</td>
</tr>
<tr>
<td>MIMC(<em>1)/( G</em>{p1}(s) )</td>
<td>8.46e-09</td>
</tr>
</tbody>
</table>

Table 7

Variances obtained with PI, PID and MIMC controllers.

For almost the same response of the tower fore-aft deflection in Fig. 9b, much better reduction of the control signal activity is obtained by the MIMC controller in Fig. 9a.

However, analyses presented until now were performed for the linear case, for the actuator \( AC= \exp(-\tau s) \). In industrial applications constraints in actuators are inevitable. In this case advantage of the proposed PID controller in Fig. 5 is its antiwindup structure.

To illustrate performance of PI, PID and MIMC controllers in the presence of rate constraints in actuator, results of simulation with and without rate constraints are compared in Fig. 10, for process \( G_{p1}(s) \) with controllers: PI\(_1\) PID\(_1\) and MIMC\(_1\).
Fig. 10 – Closed-loop system responses of nominal process \( G_p1(s) \) with controllers: \( PI_1 \) (dotted), \( MIMC_1 \) (solid) and \( PID_1 \) (dashed) for \( R(s) = 0, D(s) = 0 \) and \( N(s) = 0.3\exp(-5s)/s \). In (a) – (b) without rate constraints and in (c) – (d) with strong rate constraints: rising rate 1s\(^{-1}\) and falling rate –1s\(^{-1}\).

Finally, results of simulation with and without rate constraints are compared in Fig. 11, for process \( G_p1(s) \) with controllers: \( PID_1 \) and \( PI_{1\text{new}} \). Controller \( PI_{1\text{new}} \) is proposed here. It is obtained by applying SIMC rules to simplify the Second-Order Plus Dead-Time (SOPDT) model \( G_{\text{pid1}}(s) \) to the First-Order Plus Dead-Time (FOPDT) model \( G_{\text{pi1}}(s) \), given by

\[
G_{\text{pi1}}(s) = \frac{K e^{-\tau s}}{(Ts + 1)}, \quad T = T_1 + T_2/2, \quad \tau = \tau_{\text{pid}} + T_2/2, \quad (18)
\]

where parameters \( K, T_1, T_2 \) and \( \tau_{\text{pid}} \) are taken from the first row in Table 3.

\( PI_{1\text{new}} \) controller is implemented as in Fig. 5, with \( T_d = 0 \), time constant \( T_{fb} \) from the first row of Table 3, \( K_p \) and \( T_i, T_i \) defined by (18) and tuning rule [7]

\[
K_p = \frac{T}{2K\tau}, \quad T_i = \min\{T, 8\tau\}. \quad (19)
\]

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Results in Fig. 11 confirm that an effective anti-reset windup PI pitch controller for process $G_{p1}(s)$ can be easily obtained by the procedure proposed here.

![Graphs](a) and (b)

**Fig. 11** – Closed-loop system responses of nominal process $G_{p1}(s)$ with controllers: $PID_1$ (solid) and $PI_{1new}$ (dashed), for $R(s)$, $D(s)$ and $N(s)$ as in Fig. 10. In a) without rate constraints. In b) with strong rate constraints as in Fig. 10.

Due to space limitations, further analyses with the PI pitch controller proposed here is not presented. However, presented details makes possible to repeat some previous simulations with the $PI_{1new}$ controller, as well as to design a $PI_{2new}$ controller for process $G_{p2}(s)$. It is believed that design of PI/PID pitch controller proposed here is important for further development and comparative analyses of pitch controllers.

## 5 Conclusion

Better performance/robustness tradeoff is obtained with the MIMC and PID pitch controller, compared to PI pitch controllers from [1, 2].

It is important that inclusion of derivative action, in the presence of the simulated stochastic variation of the wind, results into performance improvement obtained with the reduced control signal activity. Better reduction of the control signal activity is obtained with the proposed MIMC controller. However, advantage of the proposed PID controller in Fig. 5 is its antiwindup structure.

Tuning of the proposed PID and PI pitch controller is simple and can be performed experimentally, by applying SIMC tuning rules to SOPDT model $G_{pid}(s)$ and FOPDT model $G_{pi}(s)$, both determined from open-loop process step responses memorized for different operating regimes.
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7 References


