

# Composite Resistor Standard for Calibration of Measuring Transducers in Laboratory Conditions

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**Abstract:** Calibration of measuring transducers for precision measurement is done by measuring voltage drop at the resistor standard, produced by output dc current proportional to the input measured value. Resistance fluctuations due to the temperature coefficient of the resistor standard are minor, thanks to the stable temperature conditions in laboratory environment. This fact brings the need to calculate the effect of resistor self-heating on its resistance. This thermal effect, produced by the flow of current through the resistor, is often disregarded. For the precise measurements this can be a significant source of error and must be quantified. This paper describes mathematical model of measurement error, resistor self-heating coefficient is defined, as it's not usually given in product datasheets. The effect on measurement results is given in detail. Composite resistor standard prototype is described, made from off-the-shelf mass produced components, calculated and hand selected to cancel the self-heating coefficient effects. The prototype is compared to the existing commercially available high performance resistor standard.

**Keywords:** Calibration, Temperature coefficient, Limit of error; Transducer, Metrology, Instrumentation, Composite resistor standard.

## 1 Introduction

Laboratory for Metrology at Faculty of Technical Sciences (FTN) is accredited for calibration of measuring transducers for ac voltage and current, resistance, true and reactive power. In the process of constant improvement of calibration performances, Laboratory tests and measures new means of reducing measurement error and measurement uncertainty.

Chair of Electrical Measurement at FTN developed several instruments for measurement of power and energy in electrical grid, and also for ac voltage and current, all based on stochastic A/D conversion. These instruments are expected to be used as standards, so they must be tested and calibrated in the Laboratory. If their measurement errors are several times lower than compared to currently used industrial equipment, they could be used as measuring standards.

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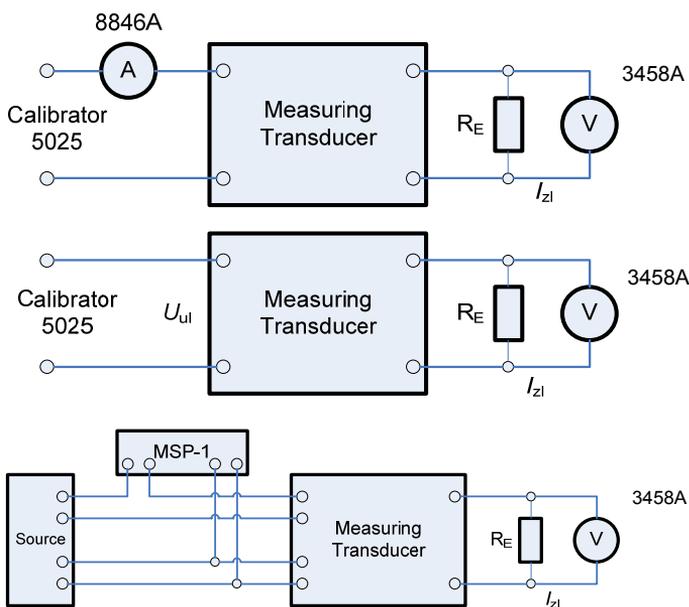
## 2 Transducer Calibration

Limit of error for measurement of ac current and power is 0.1%, and for ac voltage it is 0.02%, as defined in the Laboratory internal document [11].

Measuring transducer (MT) is a measuring device that converts a physical quantity being measured into an output signal for subsequent processing. This transformation is performed with established accuracy, while the signal at the output of a measuring transducer can't be observed directly and must be measured.

In order to test measuring transducers used as electrical standards, this error must be reduced by improvement in all stages of the measurement process.

Fig. 1 shows block diagram of calibration of measuring transducers for current, voltage and true power.



**Fig. 1** – Calibration of measuring transducers for current, voltage and true power.

Reference value is set at the input of measuring transducer under test. Best standards available in Laboratory are being used to minimize the error.

Calibrator Time Electronics 5025 is used as the ac current and voltage source. Multimeter Fluke 8846A is used as ammeter. Two-channel voltage generator high-stability standard is being used as true power source, measured with high accuracy power-meter MSP-1.

All electric measuring transducers have standardized dc current output of 0 to 10 mA or -10 to +10 mA. This current is applied to 10 Ω resistor standard  $R_E$ . Resulting dc voltage is then measured with HP 3458A multimeter, as the measurement of dc voltage is several orders of magnitude more accurate than for dc current. Voltage range is 100 mV (10 Ω 10 mA = 100 mV).

Mathematical model of calibration result error, according to [2] and [3], is given as:

$$G(X_{ul}) = K_n \frac{U_{iz} + k_{U_{iz}}}{R_E + k_{R_E}} - (X_{ul} + k_{X_{ul}}). \quad (1)$$

Measurement uncertainty  $u(x)$  of the measuring transducer calibration results in (1) can be generalized for the input value  $X_{ul}$  as:

$$u(X_{ul}) \text{ £ } \sqrt{\left(\frac{K_n}{R_E}\right)^2 u(k_{U_{iz}})^2 + \left(\frac{K_n U_{iz}}{R_E^2}\right)^2 u(k_{R_E})^2 + 1 \times u(k_{X_{ul}})^2}, \quad (2)$$

$G(X_{ul})$  – Error of calibration result;

$u(x)$  – Measurement uncertainty for value  $x$ ;

$K_n$  – Nominal transducer conversion factor;

$U_{iz}$  – Measured dc voltage value at output of transducer;

$k_{U_{iz}}$  – Correction for transducer voltage output measurement results;

$R_E$  – Resistance of resistor standard;

$k_{R_E}$  – Correction for resistor standard resistance value

$X_{ul}$  – Measured quantity value at input of transducer;

$k_{X_{ul}}$  – Correction for transducer input quantity value measurement results.

Equation (2) is given for every measuring transducer in this calibration system, where  $X_{ul}$  is the input quantity being measured.

It is obvious that the only part of the Laboratory calibration system that can be improved is resistor standard  $R_E$ . This resistor must be with high precision and stability of resistance value over time, small temperature coefficient.

Examining the measurement conditions in the Laboratory, temperature coefficient seems not very important as temperature in laboratory room is very stable, set to  $23 \pm 2^\circ\text{C}$  over a year. Thus, we can assume that ambient temperature is constant for the single set of measurements, and the Correction for resistor standard is only given for resistor value tolerance, as there is no thermal induced change in resistance at the constant temperature.

### 3 Resistor Temperature Coefficient

Effect of resistance change with temperature variation, according to [7] and [8], for resistor  $R$  at temperature  $T$ , can be approximated as:

$$R = R_0 (1 + T_C (T - T_0)), \quad (3)$$

where  $R_0$  is initial resistance value at temperature  $T_0$ , and  $T_C$  is temperature coefficient of resistance (TCR) for resistor  $R$ .

High precision, laser trimmed metal film resistors on ceramic substrate, with tolerances below 1 ppm (parts per million) are widely available today. Their TCR is low, and depending on the used technology, can be 0.05 to 10 ppm/°C, but with higher price, [7, 9].

Typical example of such resistors is VSMP2018, ultra high precision Bulk Metal Z-foil wraparound chip resistor by Vishay [10]:

- TCR typical 0.05 ppm/°C (0 to +60°C).
- Tolerance to 0.01%.
- Power rating 0.75 W (at +70°C).
- Load life stability: to  $\pm 0.005\%$  at 70°C, 2000 h at rated power.
- Price: 20 euros approx.

These specifications fit the needs for the resistor standard, as the expected measurement errors for measuring transducer are 20 to 30 ppm.

Established Laboratory procedure uses thick film resistor for  $R_E$ , with TCR of +50 ppm/°C. Even in stable temperature laboratory conditions, voltage value reading varies 4 to 5  $\mu\text{V}$ , resulting in small error  $\Gamma$  for MT calibration:

$$\Gamma = \frac{5 \mu\text{V}}{100 \text{ mV}} \cdot 100 = 0.005 \% = 50 \text{ ppm} . \quad (4)$$

This error is allowed for industrial grade MT [11], but for MT standards measurement error must be lower. This variation of resistance value is produced during the measurement process, where resistor is heated by the output current flowing through it. Joule heating (thermal losses) rise temperature of load resistor in short time, while there is no change in ambient temperature. This self-heating is unwanted by-product of measurement, so with this localized temperature variation and TCR, resistor value  $R$  changes for some value  $\Delta R$ . Metal film resistors have positive TCR and resistance increases with temperature, resulting in measurement error. Measured voltage  $U$  in this case (with positive TCR) is:

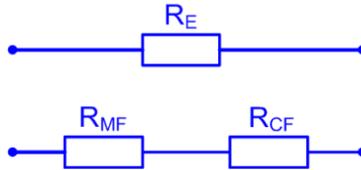
$$U = I(R + \Delta R). \quad (5)$$

This equation shows that resistance difference by self-heating cannot be avoided, because measurement itself produces the error. Answer to this problem is very-low TCR resistor standard, such as Vishay VSMP resistor.

#### 4 Composite Resistor Standard

With above analysis in mind, question could be asked: Can we produce similar results with common, cheap resistors with high TCR?

Basic idea behind this is the fact that two of the most widely available resistor types, metal film and carbon film, have opposite (negative and positive) TCRs that could be combined in such way that their TCRs are cancelled by each other. Resulting resistor is called Composite resistor standard (CRS), as a result of being made from combination of different materials. Fig. 2 shows general substitute for  $R_E$  with combination of metal and carbon film resistors with opposite TCRs.



**Fig. 2** – Common resistor standard substituted with equivalent circuit of metal and carbon film resistors in series.

Connecting several, or  $n$ , same-value resistors in parallel decreases the self-heating effect, as the divided current through each resistor is  $n$  times smaller than input current. In effect, dissipated power is  $n^2$  times smaller (10). In order to verify this theory, resulting CRS must be tested and compared with commercial low TCR resistor standard.

Carbon film resistors have negative TCR ( $T_{CCF}$ ) in range  $-200$  to  $-500$  ppm/ $^{\circ}\text{C}$ , while metal film resistors have positive TCR ( $T_{CMF}$ ) in range  $50$  to  $100$  ppm/ $^{\circ}\text{C}$ , [1, 5]. As the exact TCR of the available resistors is not known, it must be determined experimentally. Resistors in question are from the same factory batch, so we can expect uniform characteristics for each one.

Ten carbon film (CF) and ten metal film (MF) resistors, randomly chosen from two batches of 200 resistors, are placed on metal plate that is gradually heated from  $25$  to  $70^{\circ}\text{C}$ , in steps of  $5^{\circ}\text{C}$ . In every step, after 5 minutes of resistor body temperature stabilization, resistance of each resistor is measured with multimeter Fluke 8846A. Thermo paste with high thermal conductance is applied to resistor bodies for better thermal contact with metal plate.

Temperature is measured with PT temperature probe and Fluke 87 V multimeter. Then TCRs can be calculated (3), for each resistor separately, based on resistance change for  $5^{\circ}\text{C}$  increase in each step. Mean value of TCR is calculated based on TCR values for each step. As single resistor TCR is mostly constant over this temperature range, mean value is good approximation for TCR. Table 1 shows that variations of TCRs for resistors from the same group are small, so that mean value of those ten values can approximate general TCR for given resistor type. Now we have TCR for both types of resistors,  $T_{CMF}$  and  $T_{CCF}$ .

Measuring a group of resistors in ten points at different working temperatures better determinates real TCR, rather than measuring just a single resistor at one temperature [1, 7, 8].

**Table 1**  
Mean value of measured TCR for each carbon and metal film resistor, and mean value for both resistor types.

$R$	$T_{CCF}$ [ppm/°C]	$T_{CMF}$ [ppm/°C]
1	-200	50
2	-230	60
3	-270	50
4	-230	80
5	-210	70
6	-280	55
7	-250	70
8	-260	60
9	-220	40
10	-250	65
mean by type	-240	60

It must be noted that mass produced resistors can have large variations of TCR from unit to unit, but in practice [1] it is shown that resistors from the same batch or pack have very similar and uniform performance, and this assumption is verified by this experiment.

Now we can write (6) and (7) for metal and carbon film resistors connected in series that will cancel opposite TCRs in ideal case:

$$|T_{CMF}|R_{MF} - |T_{CCF}|R_{CF} = 60R_{MF} - 240R_{CF} = 0, \quad (6)$$

$$|T_{CCF}| = 4|T_{CMF}|. \quad (7)$$

Also, equivalent resistance must be:

$$R_{MF} + R_{CF} = 10 \Omega. \quad (8)$$

Finally, resistances are:

$$R_{MF} = 8 \Omega, \quad R_{CF} = 2 \Omega. \quad (9)$$

## 5 Resistor Self-heating Coefficient

Equations (6)-(9) are valid in the case of ambient temperature change, as TCR is defined for external temperature changes. As temperature is considered stable in the laboratory environment, this effect is minimal. Therefore, Self-

Heating Temperature Coefficient of Resistance (SHTCR) must be considered in order to determine resistance change by self-heating effect. This value is not given in standard datasheets by most of the manufacturers, so it must be calculated, [4 – 6].

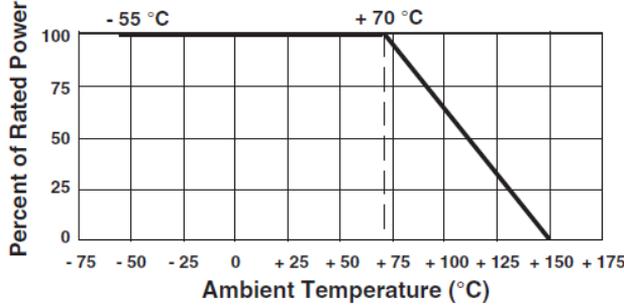
First, power dissipation at the resistor must be found. Worst case is considered, when the transducer output current is maximal, 10 mA. True power  $P$  dissipated on resistor  $R$  is:

$$P=UI=RI^2=1\text{ mW}. \quad (10)$$

While SHTCR cannot be found in datasheets, Percent of Rated Power (PRP) against ambient temperature is usually given. Fig. 3 shows typical PRP derating curve for both types, rated at 0.25 W, as modern mass produced MF and CF type resistors have similar PRP for similar nominal power, [9, 10].

The power derating curve PRP dictates the maximum power dissipation versus ambient temperature that the resistor can dissipate without exceeding the maximum specified temperature [6].

Maximum power dissipation is equal to nominal power over range 0 to 70°C, and then sharply falls to 0% at 150°C, when the resistor body material burns out and disintegrates, [6].



**Fig. 3** – Power derating curve with ambient temperature change for 0.25 W resistors.

From Fig. 3 SHTCR  $\theta_R$  can be determined:

$$\theta_R = \frac{T_{2\max} - T_{1\max}}{P_{T\max}} = \frac{150^\circ\text{C} - 70^\circ\text{C}}{0.25\text{ W}} = 320 \frac{^\circ\text{C}}{\text{W}}, \quad (11)$$

where:  $T_{1\max}$  is maximum temperature where resistor can still dissipates 100% of nominal power under normal operation,  $T_{2\max}$  is temperature where dissipation falls to 0%,  $P_{T\max}$  is nominal power dissipation at the resistor.

Available resistors were with nominal dissipation power of 0.25 W, MF resistors with 2% tolerance, and CF resistors with 5% tolerance.

Increase of resistor self-heating temperature  $\Delta T_{SZR}$  is calculated as:

$$\Delta T_{SZR} = \theta_R P_R = 320 \frac{^{\circ}\text{C}}{\text{W}} \cdot 1 \text{ mW} = 0.32 \text{ } ^{\circ}\text{C} . \quad (12)$$

Finally, we can calculate change of resistance due the self-heating:

$$\Delta R_{TSZ} = \pm R_0 \frac{T_C \Delta T_{SZR}}{1\ 000\ 000} \text{ ppm} , \quad (13)$$

where  $T_C$  is:  $T_{CMF}$  for metal film (positive TCR), and  $T_{CCF}$  for carbon film (negative TCR).

When  $n$  same-value resistors are connected in parallel, resistance change is:

$$\Delta R_n = \frac{\Delta R_{TSZ}}{n} . \quad (14)$$

Now we can calculate resistors needed for CRS to be made. As the resistances changes of MF and CF are also opposite, we want them to be equal by absolute value:

$$|\Delta R_{nMF}| = |\Delta R_{nCF}| . \quad (15)$$

CRS is made out of  $n_{MF}$  MF resistors ( $R_{nMF}$ ) in parallel, and  $n_{CF}$  CF resistors ( $R_{nCF}$ ), also in parallel, with both groups connected in series, as in Fig. 4.

Equal resistance must be the same again:

$$\frac{R_{nMF}}{n_{MF}} + \frac{R_{nCF}}{n_{CF}} = 10 \ \Omega . \quad (16)$$

From equations (13) – (15):

$$\frac{R_{nMF}}{n_{MF}} \frac{T_{CMF}}{1\ 000\ 000} \frac{\theta_R I^2 R_{nMF}}{n_{MF}^2} = \frac{R_{nCF}}{n_{CF}} \frac{T_{CCF}}{1\ 000\ 000} \frac{\theta_R I^2 R_{nCF}}{n_{CF}^2} . \quad (17)$$

Now we can determine the rule for full compensation of self-heating resistance change of CRS, as we now know TCRs for all resistors (7):

$$\frac{R_{nMF}^2}{n_{MF}^3} = \frac{4R_{nCF}^2}{n_{CF}^3} . \quad (18)$$

If we choose  $n = 6$  for number of resistor in parallel in both groups:

$$R_{nMF} = 2R_{nCF} , \quad (19)$$

and:

$$R_{nMF} + R_{nCF} = nR_E = 60 \ \Omega . \quad (20)$$

Finally, we can calculate resistors needed:

$$R_{nMF} = 40 \ \Omega = 20 \ \Omega + 20 \ \Omega , \quad R_{nCF} = 20 \ \Omega . \quad (21)$$

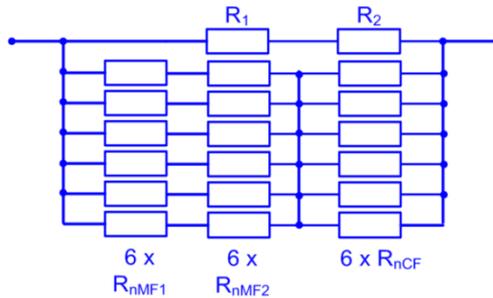
We can see that CRS could be made with only one value ( $20\ \Omega$ ) of both types of resistors.

## 6 Prototype

All resistors were manually matched to  $\pm 0.1\%$  tolerance of nominal resistance value, out of 200 pieces from the same batch. In this way, low-value tolerance of final CRS is obtained [1], using standard resistors of 2% and 5% tolerance. Resistors are matched and measured in 4-wire mode with Fluke 8846A multimeter, at  $23^\circ\text{C}$ .

Fig. 4 shows details of CSR schematic. Resistors  $R_1$  (CF) and  $R_2$  (MF) are used for fine trimming of total resistance to accurate value ( $10\ \Omega$ ). They have much larger value than  $R_E$  (in order of  $100\ \text{k}\Omega$ ), so the current flowing through this line is very small (in order of  $\mu\text{A}$ ), so the dissipated power on them is negligible.

As the self-heating effect is not affecting them, ratio of  $R_1$  and  $R_2$  resistances is chosen per (6) and (7) to cancel any possible resistance change due variations in ambient temperature.



**Fig. 4** – Equivalent circuit of Composite resistor standard.

Fig. 5 shows external view of CSR prototype MB-10R, realized with above calculations. External connectors are used for 4-wire sensing (Kelvin method).

Fig. 6 shows internal view of CSR prototype.

Great care must be taken to avoid overheating of resistors during soldering. Carbon film resistors can permanently change their resistance value due the high temperature, while metal film resistors tend to regain their nominal resistance after period of cooling. This can offset the calculated values, and some additional resistors might be needed to correct the nominal CRS value, as evident in Fig. 6.

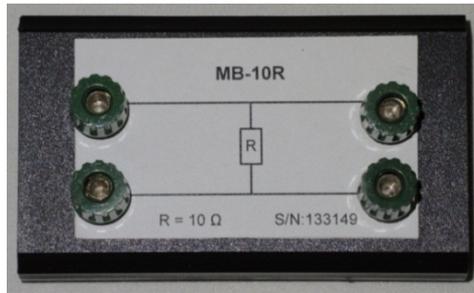


Fig. 5 – External view of CSR prototype *MB-10R*.



Fig. 6 – Internal view of CSR prototype *MB-10R*.

## 7 Measurement Results

Finished CRS prototype is measured with HP 3458A multimeter. Measurements were performed on regular monthly basis from March, 2013 to September, 2015. Results are given in Fig. 7, as nominal resistance value variations (in ppm) over time.

Final overview of CSR MB-10R data:

- Nominal value is  $R_0 = 10.00021 \Omega$  (at  $23^\circ\text{C}$ ), resulting in tolerance of  $+0.0021 \%$  (calculated as mean value from the Fig. 7). Value varied in range of  $21 \pm 6$  ppm.
- TCR is  $0.02 \text{ ppm}/^\circ\text{C}$  (at  $23$  to  $70^\circ\text{C}$ ), with possibility that real value is even lower, but cannot be expressed using the instruments available to Laboratory.
- Maximum dissipated power:

$$P_{RE} = R_E I_{RE \max}^2 = R_E (6 I_{RnMF \max})^2 = 360 \frac{P_{T \max}}{R_{nMF}}, \quad (22)$$

$$P_{RE} = 2.25 \text{ W}.$$



The self-heating temperature coefficient must be calculated as it is not given in datasheets. The effect of resistance change due self-heating is explained, and it must be considered during high-precision measurements, as it can affect measurement error and uncertainty.

For measurements with high current values, larger impact of self-heating is expected as the result of larger power dissipation, proportional to squared value of measured current.

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