A Modified ABC Algorithm for Solving Optimal Power Flow Problem

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Abstract: This paper presents a modified artificial bee colony (MABC) algorithm for solving the optimal power flow (OPF) problem in power system. Artificial bee colony algorithm is a recent population-based optimization method which has been successfully used in many complex problems. A new mutation strategy inspired from the differential evolution (DE) is introduced in order to improve the exploitation process. The new algorithm is implemented to the OPF problem so as to minimize the total generation cost when considering the equality and inequality constraints. In order to validate of the proposed algorithm, it is applied to the standard IEEE 30-bus test system. The results show that the proposed technique provides better solutions than other heuristic techniques reported in literature.

Keywords: Modified artificial bee colony algorithm, Differential evolution, Optimization, Optimal power flow, Power system.

1 Introduction

Power systems are becoming increasingly more complex due to the interconnection of regional system, deregulation of the overall electricity market and increased in power demand. So power engineers are looking for ways to better utilize their existing transmission systems. The optimal power flow (OPF) problem has become an essential for operation, planning and control of power systems. It was proposed first time in 1968 by Dommel and Tinney [1]. The main goal of OPF problem is to optimize a selected objective function such as fuel cost, power loss etc. In solving OPF problem, objective function is optimized by adjusting system control variable while satisfying the equality constraints and inequality constraints. The equality constraints are the power flow equations, while the inequality constraints are the limits on control variables and the operating limits of the power system dependent variables. Many conventional techniques such as gradient-based method, Newton method, linear programming, and quadratic programming have been employed for the solution of OPF problem [2–4]. But these methods cannot find a global

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optimization solution in OPF problems which have nonlinear constraints and objective function. Recently, numerous heuristic algorithms have been developed and have been implemented to successfully generate OPF solution such as tabu search (TS) [5], genetic algorithm (GA) [6–8], evolutionary programming (EP) [9], artificial bee colony (ABC) algorithm [10–12], differential evolution (DE) [13–16], teaching learning-based optimization (TLBO) [17–19], biogeography-based optimization (BBO) [20–22], particle swarm optimization (PSO) [23–25], and gravitational search algorithm (GSA) [26].

Swarm intelligence has become a research interest to different domain of researchers in recent years. These algorithms simulate the food foraging behavior of a flock of birds or swarm of bees. Motivated by the foraging behavior of honeybees, researchers have initially proposed artificial bee colony (ABC) algorithm for solving various optimization problems [27, 28]. Artificial bee colony (ABC) algorithm is a relatively new member of swarm intelligence. ABC tries to model natural behavior of real honey bees in food foraging. Honey bees use several mechanisms like waggle dance to optimally locate food sources and to search new ones. This makes them a good candidate for developing new intelligent search algorithms. Despite the simplicity and the superiority of ABC algorithm, recent studies reported that it suffers from a poor exploitation process and a slow convergence rate. To overcome these pitfalls, some research papers have introduced modifications to the classical ABC algorithm in order to improve its performance and tackle more complex real-world problems [29, 30].

In this paper, an efficient approach is proposed to solve the OPF problems using MABC technique. The performance of the proposed approach has been demonstrated on the standard IEEE 30-bus test system. Simulation results demonstrate that the proposed method provides better results than other heuristic techniques.

2 Problem Formulation

The optimal power flow problem solution aims to optimize a selected objective function via optimal adjustment of the power system control variables, while at the same time satisfying various equality and inequality constraints. Generally, the OPF problem can be mathematically written as follows:

$$\text{Min } J(x,u),$$

subject to

$$g(x,u) = 0,$$
$$h(x,u) \leq 0,$$
where \( J \) is objective function to be minimized, \( g \) is the equality constraints represent typical load flow equations, \( h \) is the inequality constraints represent the system operating constraints, \( x \) is the vector of dependent variables or state vector consisting of:

- Active power of generators at slack bus \( P_{G1} \).
- Load bus voltage \( V_L \).
- Generator reactive power output \( Q_G \).
- Transmission line loading (line flow) \( S_i \).

Hence, \( x \) can be expressed as:

\[
x^T = [P_{G1}, V_{L1}, \ldots, V_{NL}, Q_{G1}, \ldots, Q_{NNG}, S_{1l}, \ldots, S_{nl}],
\]

where \( N_G, N_L, \) and \( n_l \) are the number of generators, number of load buses, and number of transmission lines, respectively. \( u \) is the vector of independent variables or control variables consisting of:

- Generator voltage \( V_G \) at PV bus.
- Generator real power output \( P_G \) at PV buses except at the slack bus \( P_{G1} \).
- Transformer tap setting \( T \).
- Shunt VAR compensation (or reactive power of switchable VAR sources) \( Q_c \).

Hence, \( u \) can be expressed as:

\[
u^T = [P_{G2}, \ldots, P_{NNG}, V_{G1}, \ldots, V_{NNG}, Q_{c1}, \ldots, Q_{cNC}, T_1, \ldots, T_{NT}]
\]

where \( N_T \) and \( N_C \) are the number of the regulating transformer and VAR compensators, respectively.

### 2.1 Objective function

The objective function for the OPF reflects the cost associated with generation in power system. The objective function for the whole power system can then be written as the sum of the fuel cost model for each generator:

\[
J = \sum_{i=1}^{NG} F_i ,
\]

where \( F_i \) indicate the fuel cost of the \( i \)-th generator.

The fuel cost curve for any unit is assumed to be approximated by segments of quadratic functions of the active power output of the generator as:

\[
F_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2 ,
\]

where \( a_i, b_i, \) and \( c_i \) are the cost coefficient of the \( i \)-th generator, \( P_{Gi} \) is the power generated by the \( i \)-th unit and \( N_G \) is the number of generators.
2.2 The equality constraints

These constraints are specific load flow equations which can be described as follows:

\[
P_{Gi} - P_{Di} - V_i \sum_{j=1}^{NB} V_j \left[ G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j) \right] = 0, \tag{7}
\]

\[
Q_{Gi} - Q_{Di} - V_i \sum_{j=1}^{NB} V_j \left[ G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j) \right] = 0, \tag{8}
\]

where, \( i = 1, \ldots, NB \), \( NB \) is the number of buses; \( P_G \) is the active power generated, \( Q_G \) is the reactive power generated, \( P_D \) is the load active power, \( Q_D \) is the load reactive power, \( G_{ij} \) and \( B_{ij} \) respectively indicate the real part and imaginary part of the \( ij \)-th element of the node admittance matrix.

2.3 The inequality constraints

These constraints reflect the system operating limits as follows:

1. Generator constraints: generator voltages, real power outputs, and reactive power outputs are restricted by their lower and upper limits as follows:

\[
V^\text{min}_{Gi} \leq V_{Gi} \leq V^\text{max}_{Gi}, \quad i = 1, \ldots, NG, \tag{9}
\]

\[
P^\text{min}_{Gi} \leq P_{Gi} \leq P^\text{max}_{Gi}, \quad i = 1, \ldots, NG, \tag{10}
\]

\[
Q^\text{min}_{Gi} \leq Q_{Gi} \leq Q^\text{max}_{Gi}, \quad i = 1, \ldots, NG; \tag{11}
\]

2. The transformer constraints: transformer tap settings are bounded as follows:

\[
T^\text{min}_i \leq T_i \leq T^\text{max}_i, \quad i = 1, \ldots, NT; \tag{12}
\]

3. Shunt VAR constraints: shunt VAR compensations are qualified by their limits as follows:

\[
Q^\text{min}_{ci} \leq Q_{ci} \leq Q^\text{max}_{ci}, \quad i = 1, \ldots, NC; \tag{13}
\]

4. Security constraints: these include the constraints of voltages at load busses and transmission line loadings as follows:

\[
V^\text{min}_{Li} \leq V_{Li} \leq V^\text{max}_{Li}, \quad i = 1, \ldots, NL, \tag{14}
\]

\[
S^\text{min}_{li} \leq S_{li} \leq S^\text{max}_{li}, \quad i = 1, \ldots, nl. \tag{15}
\]

3 Overview of Artificial Bee Colony (ABC) Algorithm

Artificial bee colony is one of the most recently defined algorithms by Karaboga, motivated by the intelligent behavior of honey bees [27, 28]. In the ABC system, artificial bees fly around in the search space, and some (employed
and onlooker bees) choose food sources depending on the experience of themselves and their nest mates, and adjust their positions. Some (scouts) fly and choose the food sources randomly without using experience. If the nectar amount of a new source is higher than that of the previous one in their memory, they memorize the new position and forget the previous one. Thus, the ABC system combines local search methods, carried out by employed and onlooker bees, with global search methods, managed by onlookers and scouts, attempting to balance exploration and exploitation process.

In the ABC algorithm, the colony of artificial bees consists of three groups of bees: employed bees, onlooker bees, and scout bees. The main steps of the ABC algorithm are described as follows:

– INITIALIZE.
– REPEAT.
  (a) Place the employed bees on the food sources in the memory;
  (b) Place the onlooker bees on the food sources in the memory;
  (c) Send the scouts to the search area for discovering new food sources;
  (d) Memorize the best food source found so far.
– UNTIL (requirements are met).

In the ABC algorithm, each cycle of the search consists of three steps: moving the employed and onlooker bees onto the food sources, calculating their nectar amounts respectively, and then determining the scout bees and moving them randomly onto the possible food source. Here, a food source stands for a potential solution of the problem to be optimized. The ABC algorithm is an iterative algorithm, starting by associating all employed bees with randomly generated food solutions. The initial population of solutions is filled with $SN$ number of randomly generated $D$ dimensions. Let $X_i = \{x_{i1}, x_{i2}, \ldots, x_{iD}\}$ represent the $i$th food source in the population, $SN$ is the number of food source equal to the number of the employed bees and onlooker bees. $D$ is the number of optimization parameters. Each employed bee $x_{ij}$ generates a new food source $v_{ij}$ in the neighborhood of its currently associated food source by (16), and computes the nectar amount of this new food source as follows:

$$v_{ij} = x_{ij} + \phi_{ij} \left( x_{ij} - x_{ik} \right), \quad (16)$$

where $\phi_{ij} = (rand - 0.5) \times 2$ is a uniformly distributed real random number within the range [-1, 1], $i \in \{1,2,\cdots,SN\}$, $k = \text{int} \left( rand \times SN \right) + 1$ and $k \neq 1$, and $j \in \{1,2,\cdots,D\}$ are randomly chosen indexes. The new solution $v_i$ will be accepted as a new basic solution, if the objective fitness of $v_i$ is smaller than the fitness of $x_i$, otherwise $x_i$ would be obtained.
When all employed bees finish this process, an onlooker bee can obtain the information of the food sources from all employed bees and choose a food source according to the probability value associated with the food source, using the following expression:

\[ p_i = \alpha \times \frac{fit_i}{\text{max}(fit)} + \beta; \quad \alpha + \beta = 1, \]  

(17)

where \( fit_i \) is the fitness value of the solution \( i \) evaluated by its employed bee. Obviously, when the maximum value of the food source decreases, the probability with the preferred source of an onlooker bee decreases proportionally. Then the onlooker bee produces a new source according to (16). The new source will be evaluated and compared with the primary food solution, and it will be accepted if it has a better nectar amount than the primary food solution.

After all onlookers have finished this process, sources are checked to determine whether they are to be abandoned. If the food source does not improve after a determined number of the trail “limit”, the food source is abandoned. Its employed bee will become a scout and then will search for a food source randomly as follows:

\[ x_{ij} = x_{j\text{min}} + \text{rand}(0, 1) \times (x_{j\text{max}} - x_{j\text{min}}), \]  

(18)

where \( x_{j\text{min}} \) and \( x_{j\text{max}} \) are lower and upper bounds for the dimension \( j \) respectively.

After the new source is produced, another iteration of the ABC algorithm will begin. The whole process repeats again till the termination condition is met.

4 Modified Artificial Bee Colony (MABC) Algorithm

Following this spirit, a modified ABC algorithm inspired from DE to optimize the objective function of the OPF problems. DE is an evolutionary algorithm first introduced by Storn and Price [31, 32]. Similar to other evolutionary algorithms, particularly genetic algorithm, DE uses some evolutionary operators like selection recombination and mutation operators. Different from genetic algorithm, DE uses distance and direction information from the current population to guide the search process. The crucial idea behind DE is a scheme for producing trial vectors according to the manipulation of target vector and difference vector. If the trail vector yields a lower fitness than a predetermined population member, the newly trail vector will be accepted and be compared in the following generation. Currently, there are several variants of DE. The particular variant used throughout this investigation is the DE/rand/1 scheme. The differential mutation strategy is described by the following equation:
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\[ v_i = x_a + F (x_b - x_c), \]

where \( a, b, c \in SN \) are randomly chosen and mutually different and also different from the current index \( i \). \( F \in (0, 1) \) is constant called scaling factor which controls amplification of the differential variation of \( x_{bj} - x_{cj} \).

Based on DE and the property of ABC algorithm, modification of the search solution is explained by (20) as follows:

\[ v_{ij} = x_{aj} + \phi_{ij} (x_{ij} - x_{bj}), \]

(20)

The new search method can generate the new candidate solutions only around the random solutions of the previous iteration.

Akay and Karaboga [29] proposed a modified artificial bee colony algorithm by controlling the frequency of perturbation. Inspired by this algorithm, the control parameter, i.e., modification rate (\( MR \)) is used. In order to produce a candidate food position \( v_{ij} \) from the current memorized \( x_{ij} \), improved ABC algorithm uses the following expression [30]:

\[ v_{ij} = \begin{cases} x_{aj} + \phi_{ij} (x_{ij} - x_{bj}), & \text{if } R_{ij} \leq MR, \\ x_{ij}, & \text{otherwise}, \end{cases} \]

(21)

where \( R_{ij} \) is a uniformly distributed real random number within the range \([0, 1]\).

The pseudo-code of the modified ABC algorithm is given below:

---

Initialize the population of solutions \( x_{ij}, i = 1, \ldots, SN; j = 1, \ldots, D, \text{ trial}_i = 0; \text{ trial}_i \) is the non-improvement number of the solution \( x_i \), used for abandonment.

Evaluate the population.

\( \text{cycle} = 1 \)

repeat

\{ --- Produce a new food source population for employed bee --- \}

for \( i = 1 \) to \( SN \) do

Produce a new food source \( v_i \) for the employed bee of the food source \( x_i \) by using (21) and evaluate its quality:

Select randomly \( a \neq b \neq i \)

\[ v_{ij} = \begin{cases} x_{aj} + \phi_{ij} (x_{ij} - x_{bj}), & \text{if } R_{ij} \leq MR, \\ x_{ij}, & \text{otherwise}. \end{cases} \]

Apply a greedy selection process between \( v_i \) and \( x_i \) and select the better one. If solution \( x_i \) does not improve \( \text{trial}_i = \text{trial}_i + 1 \), otherwise \( \text{trial}_i = 0 \)

end for

Calculate the probability values \( p_i \) by (17) for the solutions using fitness values:

\[ p_i = \alpha \times \frac{\text{fit}_i}{\max(\text{fit}_i)} + \beta; \quad \alpha + \beta = 1 \]
{--- Produce a new food source population for onlooker bee ---}

\[
t = 0, \quad i = 1 
\]

repeat

if random \(<\ p\_i\) then

Produce a new \(v_{ij}\) food source by (21) for the onlooker bee:

Select randomly \(a \neq b \neq i\)

\[
v_{ij} = \begin{cases} 
    x_{aj} + \phi_{ij} (x_{ij} - x_{bj}), & \text{if } R_{ij} \leq MR, \\
    x_{ij}, & \text{otherwise}.
\end{cases}
\]

Apply a greedy selection process between \(v_i\) and \(x_i\) and select the better one. If solution \(x_i\) does not improve \(trial_i = trial_i + 1\), otherwise \(trial_i = 0\)

\(t = t + 1\)

end if

until \((t = SN)\)

{--- Determine scout bee ---}

if \(\max (trial_i) > \text{limit}\) then

Replace \(x_i\) with a new randomly produced solution by (18)

\[
x_{ij} = x_{j_{\text{min}}} + \text{rand}(0, 1) \times (x_{j_{\text{max}}} - x_{j_{\text{min}}})
\]

end if

Memorize the best solution achieved so far

\(\text{cycle} = \text{cycle} + 1\)

until \((\text{cycle} = \text{Maximum Cycle Number})\)

5 Simulation Results

The proposed MABC technique has been tested on the standard IEEE 30-bus test system is shown in Fig. 1. The system consists of 41 transmission lines, 6 generating unit and 4 tap-changing transformers. The complete system data is given in [33, 34]. The upper and lower active power generating limits and the fuel cost coefficients of all generators of the standard IEEE 30-bus test system are presented in Table 1.

<table>
<thead>
<tr>
<th>Bus</th>
<th>(P_{Gi}^{\text{min}}) [MW]</th>
<th>(P_{Gi}^{\text{max}}) [MW]</th>
<th>(a_i) [$/h]</th>
<th>(B) [$/MWh]</th>
<th>(c_i) [$/MW^2\text{h}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>200</td>
<td>0.00</td>
<td>2.00</td>
<td>0.00375</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>80</td>
<td>0.00</td>
<td>1.75</td>
<td>0.01750</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>50</td>
<td>0.00</td>
<td>1.00</td>
<td>0.06250</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>35</td>
<td>0.00</td>
<td>3.25</td>
<td>0.00834</td>
</tr>
<tr>
<td>11</td>
<td>10</td>
<td>30</td>
<td>0.00</td>
<td>3.00</td>
<td>0.02500</td>
</tr>
<tr>
<td>13</td>
<td>12</td>
<td>40</td>
<td>0.00</td>
<td>3.00</td>
<td>0.02500</td>
</tr>
</tbody>
</table>
The voltage magnitude limits are between 0.95 and 1.05 pu for all load buses, while it is between 0.95 and 1.1 pu for all generator buses. Tap setting of all transformer taps are between 0.9 and 1.1 pu. The total system demand was chosen 283.4 MW. Simulations were performed in MATLAB R2015a environment on a PC with a 3 GHz processor. The values of MABC algorithm for solving OPF problem in this paper are designated as follow:

The number of colony size, $NP = 20$; the number of cycles for aging, $maxCycle = 200$; the number of variables, $NV = 6$; and $limit = 100$.

In this paper, a set of control variables is formed and the formulation with OPF is solved using MABC algorithm along with 15 control variables. In this, first 5 control variables are regarded as real power generators other than slack bus generator, next 6 control variables are bus voltage magnitudes of generator and last 4 control variables are transformer-tap settings. The optimal values of control variables are given in Table 2. The total fuel cost obtained by proposed technique is 800.8622 $/h.

### Table 2

**Optimal values of control variables.**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Limit</th>
<th>Result (Best solution)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{G1}$ [MW]</td>
<td>Lower = 50, Upper = 200</td>
<td>176.8602</td>
</tr>
<tr>
<td>$P_{G2}$ [MW]</td>
<td>Lower = 20, Upper = 80</td>
<td>48.1851</td>
</tr>
<tr>
<td>$P_{G5}$ [MW]</td>
<td>Lower = 15, Upper = 50</td>
<td>21.0441</td>
</tr>
<tr>
<td>$P_{G8}$ [MW]</td>
<td>Lower = 10, Upper = 35</td>
<td>21.3692</td>
</tr>
<tr>
<td>$P_{G11}$ [MW]</td>
<td>Lower = 10, Upper = 30</td>
<td>12.5976</td>
</tr>
<tr>
<td>$P_{G13}$ [MW]</td>
<td>Lower = 12, Upper = 40</td>
<td>12.4283</td>
</tr>
<tr>
<td>$V_{G1}$ [pu]</td>
<td>Lower = 0.95, Upper = 1.10</td>
<td>1.0807</td>
</tr>
<tr>
<td>$V_{G2}$ [pu]</td>
<td>Lower = 0.95, Upper = 1.10</td>
<td>1.0619</td>
</tr>
<tr>
<td>$V_{G5}$ [pu]</td>
<td>Lower = 0.95, Upper = 1.10</td>
<td>1.0277</td>
</tr>
<tr>
<td>$V_{G8}$ [pu]</td>
<td>Lower = 0.95, Upper = 1.10</td>
<td>1.0315</td>
</tr>
<tr>
<td>$V_{G11}$ [pu]</td>
<td>Lower = 0.95, Upper = 1.10</td>
<td>1.1000</td>
</tr>
<tr>
<td>$V_{G13}$ [pu]</td>
<td>Lower = 0.95, Upper = 1.10</td>
<td>1.0624</td>
</tr>
<tr>
<td>$T_{4,12}$ [pu]</td>
<td>Lower = 0.90, Upper = 1.10</td>
<td>1.1000</td>
</tr>
<tr>
<td>$T_{6,9}$ [pu]</td>
<td>Lower = 0.90, Upper = 1.10</td>
<td>0.9000</td>
</tr>
<tr>
<td>$T_{6,10}$ [pu]</td>
<td>Lower = 0.90, Upper = 1.10</td>
<td>0.9937</td>
</tr>
<tr>
<td>$T_{27,28}$ [pu]</td>
<td>Lower = 0.90, Upper = 1.10</td>
<td>0.9657</td>
</tr>
<tr>
<td>$J [$/h]</td>
<td></td>
<td>800.8622</td>
</tr>
<tr>
<td>$P_{loss} [MW]$</td>
<td></td>
<td>9.0843</td>
</tr>
</tbody>
</table>

**Table 3** shows a comparison between the results of fuel cost and power losses obtained from the proposed approach and those reported in the literature. The comparison is performed with the same control variable limits, initial conditions, and other system data. It is clear that the proposed MABC technique outperforms TS, TLBO, BBO, ABC, and GA techniques.
Table 3
Results of minimum fuel cost for IEEE 30-bus system.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{G1}$ (MW)</td>
<td>176.04</td>
<td>176.94</td>
<td>171.9231</td>
<td>180.5218</td>
<td>177.280</td>
<td>176.8602</td>
</tr>
<tr>
<td>$P_{G2}$ (MW)</td>
<td>48.75</td>
<td>49.02</td>
<td>48.8394</td>
<td>48.7845</td>
<td>48.817</td>
<td>48.1851</td>
</tr>
<tr>
<td>$P_{G8}$ (MW)</td>
<td>22.05</td>
<td>21.81</td>
<td>21.7629</td>
<td>18.6469</td>
<td>21.810</td>
<td>21.3692</td>
</tr>
<tr>
<td>$P_{G11}$ (MW)</td>
<td>12.44</td>
<td>12.20</td>
<td>12.1831</td>
<td>11.8145</td>
<td>11.325</td>
<td>12.5976</td>
</tr>
<tr>
<td>$P_{G13}$ (MW)</td>
<td>12.00</td>
<td>11.41</td>
<td>16.5588</td>
<td>12.1011</td>
<td>12.087</td>
<td>12.4283</td>
</tr>
<tr>
<td>Fuel cost [$/h]</td>
<td>802.29</td>
<td>802.45</td>
<td>802.717</td>
<td>802.1649</td>
<td>802.0012</td>
<td>800.8622</td>
</tr>
</tbody>
</table>

Fig. 1 – IEEE 30-bus test system [23].
6 Conclusion

In this paper, a modified ABC algorithm has been applied to solve the OPF problem in power system. The proposed MABC algorithm employs a new mutation strategy inspired from the differential evolution (DE) to enhance the performance of the conventional ABC algorithm. The differential mutation is devised to improve the global searching capability and to enhance the capability of escaping from a local minimum. The proposed technique has provided the global solution in the standard IEEE 30-bus test system and it has produced results which better than the previous studies reported in literature. The obtained results indicate that the proposed technique can be effectively used to solve the optimal power flow problem.

7 References


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