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Distortion of Electric Field Homogeneity Between Two Thin Toroidal Electrodes by a Dielectric Sphere*

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Abstract: We assess the influence of a radius and a relative permittivity of an isotropic dielectric sphere on electric field homogeneity. The homogenous electric field is generated using two thin toroidal electrodes, charged by equal charges of opposite signs. The Poisson equation is solved by a method of separation of variables. Increase in a relative permittivity of the sphere and in its radius, produces more distortions of the electric field homogeneity.

Keywords: Dielectric sphere, Electrostatic, Homogeneous electrostatic field, Toroidal electrodes.

1 Introduction

A structure that creates a uniform electric field is an integral part of many modern devices and applications. Electrospinning is a low-cost method which is used in production of nanofibers. Uniform electric field enables multi-needles electrospinning head based on trapezoid arrangement [1]. A uniform electric field is also required for the detection of high-voltage power lines by surface-integrated electric field sensors [2]. Homogeneous electric field is a part of defibrillators, electrochemotherapy, electrophoresis in biochemistry etc. Some applications require not only homogeneous, but also zero electric field in the region of interest. For example, in order to prevent heating during the Magnetic Resonance Tomography (MRT), electric field-free zones should be generated in the body [3].

The problem of generating uniform electric and magnetic fields dates back to the middle of the last century [4, 5]. The simplest design for creating uniform electric field in a limited central space includes a pair of charged rings [6].

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However, a greater field homogeneity in a larger central volume can be provided by various advanced configurations, including those that are based on a usage of toroidal and biconical electrodes [7].

Once created, a homogeneous electric field can be easily disrupted. For example, an occurrence of thermally or electrically induced bubbles in the liquid medium or a presence of imperfections in the vacuum, can significantly change the homogeneity of the electric field in the vicinity of these imperfections [8, 9]. Such imperfections are commonly referred to as external bodies and they can be modeled using various geometries, such as a sphere, ellipsoid or cylinder, and various media, like conductor, bi-isotropic, isotropic or dielectric. Precise modelling of the system used for homogeneous electric field generation including the influence of external body on the field homogeneity, is of exceptional importance for a proper operation of appliances or sensors that require this field.

Several configurations for homogeneous field generation and an influence of various external bodies on a field homogeneity have been already analyzed. A system consisting of plan-parallel electrodes and an external cylindrical body of perfectly conductive, isotropic dielectric and bi-isotropic material are studied in [10 – 12]. Toroidal electrodes for homogeneous field generation and disruption of a homogeneity by a spherical conducting body and a prolate and an oblate ellipsoidal electrode are studied in [13, 14], respectively. The influence of the Tellegen type [15] bi-isotropic sphere on a homogeneous field, produced by two toroidal electrodes, is analyzed in [16].

In the paper at hand we study the influence of a radius and a relative permittivity of a dielectric isotropic sphere on the electric field homogeneity generated by two toroidal electrodes. A part of these results is presented in [17]. The paper is organized as follows: description of the model and mathematical background of solving the Poisson's potential equation are presented in Section 2. Numerical results and discussion are given in Section 3, followed by conclusions in Section 4.

2 Model and Method

A primary cell of the first order consists of two toroidal electrodes of a radius a and a distance between them $2h$ (Fig. 1). This cell is considered as the simplest configuration for generating a homogeneous electric field. Cells of a higher order can provide a higher level of homogeneity in a larger central volume, but the influence of a radius and a relative permittivity of a dielectric sphere on a filed homogeneity can already be assessed in the field of a primary cell.

The dimensions of the primary cell of the first order, $h / R = 0.77459667$ and $a / R = 0.63245552$, have been taken from [7], where a modelling of a system is done based on the Taylor series. A distance between the center of the system and each toroidal electrode is denoted by R, $R=\sqrt{a^2+h^2}$. A center of the external sphere coincides with the coordinate origin. Electric field is observed along athe $z-$axis of the system.

Calculation of an electric field around the external body placed in the homogeneous electric field can be done by a simple method of images [7, 10, 13, 14]. Using a spherical coordinate system, Fig. 1, the potential equation can be solved by the method of separation of variables in this system [18].

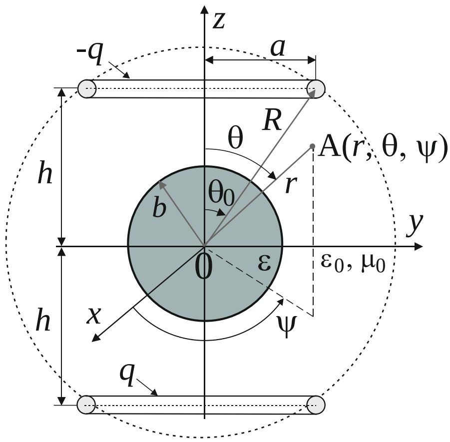


Fig. 1 – *Primary cell of the first order and a spherical external dielectric body, placed in the centre of the system.*

The constitutive relation between the electric field vector $\boldsymbol{E}$ and the vector of electric displacement field $\boldsymbol{D}$ in the case of an isotropic homogeneous medium is given by [17]:

$$\boldsymbol{D}=\varepsilon\boldsymbol{E}\,, \tag{1}$$

where $\varepsilon=\varepsilon_r\varepsilon_0$ (ε_r is a relative permittivity and $\varepsilon_0=8.8541878\cdot 10^{-12}$ F/m is the permittivity of vacuum).

Starting from the first Maxwell's equation, using (1) and assuming $\varepsilon=\text{const}$.

$$\operatorname{div}\boldsymbol{D}=\rho=\varepsilon\operatorname{div}\boldsymbol{E} \tag{2}$$

one obtains:

$$\operatorname{div}(\boldsymbol{E})=\frac{\rho}{\varepsilon}, \tag{3}$$

where ρ is the volumetric density of free charges in the observed medium.

Since the electric field in this case is irrotational, it can be determined using an electric scalar potential φ:

$$\boldsymbol{E} = -\mathrm{grad}(\varphi). \tag{4}$$

Now, the Poisson's potential equation is:

$$\Delta\phi = -\rho/\varepsilon. \tag{5}$$

The equation (5) is solved by the method of separation of variables in spherical coordinates. The resulting electric scalar potential can be presented by the following expressions:

$$\varphi = \begin{cases} \sum\limits_{n=0}^{\infty}\left(A_n r^n + \dfrac{B_n}{r^{n+1}}\right)G(\theta), & r \le b, \\ \sum\limits_{n=0}^{\infty}\left(C_n r^n + \dfrac{D_n}{r^{n+1}}\right)G(\theta), & b \le r \le R, \\ \sum\limits_{n=0}^{\infty}\left\{\left[C_n - \dfrac{q\,G(\theta_0)}{4\pi\varepsilon_0 R^{n+1}}\right]r^n + \left[D_n + \dfrac{q\,G(\theta_0)}{4\pi\varepsilon_0}R^n\right]\dfrac{1}{r^{n+1}}\right\}G(\theta), & R \le r, \end{cases} \tag{6}$$

where $G(\theta) = P_n(\cos\theta) - P_n(-\cos\theta)$. $P_n(\cos\theta)$ is a Legendre polynomial of the first kind and θ_0 is the angle between R and the $z-$axis, $\theta_0 = \arctan[a/h]$.

The unknown constants A_n, B_n, C_n and D_n have to be determined. First, using the condition that the electric scalar potential has to be finite at $r = 0$, the unknown constant B_n is determined:

$$B_n = 0. \tag{7}$$

The remaining constants are obtained satisfying the continuity of the tangential component of electric field, normal component of electric displacement field and electric scalar potential at the boundaries between two dielectrics. These constants are:

$$A_n = A_n(n) = \frac{q\,(2n+1)}{4\pi\left[n\varepsilon + (n+1)\varepsilon_0\right]R^{n+1}}G(\theta_0), \tag{8}$$

$$C_n = C_n(n) = \frac{q}{4\pi\varepsilon_0 R^{n+1}}G(\theta_0) \tag{9}$$

and

$$D_n = D_n(n) = \frac{q\,n(\varepsilon_0 - \varepsilon)b^{2n+1}}{4\pi\varepsilon_0\left[n\varepsilon + (n+1)\varepsilon_0\right]R^{n+1}}G(\theta_0). \tag{10}$$

Putting the constants $(7)-(10)$ into (6), the electric scalar potential becomes:

$$\varphi=\begin{cases}\dfrac{q}{4\pi\varepsilon_0}\displaystyle\sum_{n=0}^{\infty}\frac{2n+1}{\left(n\varepsilon+(n+1)\varepsilon_0\right)}\frac{r^n}{R^{n+1}}G(\theta_0)G(\theta), & r\le b,\\ \dfrac{q}{4\pi\varepsilon_0}\displaystyle\sum_{n=0}^{\infty}\left[\frac{r^n}{R^{n+1}}+\frac{n(\varepsilon_0-\varepsilon)}{\left(n\varepsilon+(n+1)\varepsilon_0\right)}\frac{b^{2n+1}}{R^{n+1}r^{n+1}}\right]G(\theta_0)G(\theta), & b\le r\le R,\\ \dfrac{q}{4\pi\varepsilon_0}\displaystyle\sum_{n=0}^{\infty}\left[\frac{R^n}{r^{n+1}}+\frac{n(\varepsilon_0-\varepsilon)}{\left(n\varepsilon+(n+1)\varepsilon_0\right)}\frac{b^{2n+1}}{R^{n+1}r^{n+1}}\right]G(\theta_0)G(\theta), & R\le r,\end{cases}\quad(11)$$

In this case of an isotropic dielectric, a simple method of images can be applied. However, the method of images cannot handle anisotropy and bi-isotropy. The method of separation of variables can be applied to isotropic, anisotropic and bi-isotropic dielectrics and therefore is our method of choice.

3 Numerical Results

3.1 Determination of a number of terms in series that assures satisfactory accuracy

We have performed numerical experiments in order to get a number of terms in (11) which assures that a change of a convergent sum becomes negligibly small. In this case, it is enough to take first 15 nonzero terms of the series in order to get a satisfactory level of accuracy. By taking into account additional terms, the electric scalar potential is almost not affected.

Coefficients A_n, C_n and D_n defined by (8)-(10), for the case of b/R = 0.4 ε_r = 0.2 and ε_r = 15 are presented in Figs. 2 and 3, respectively.

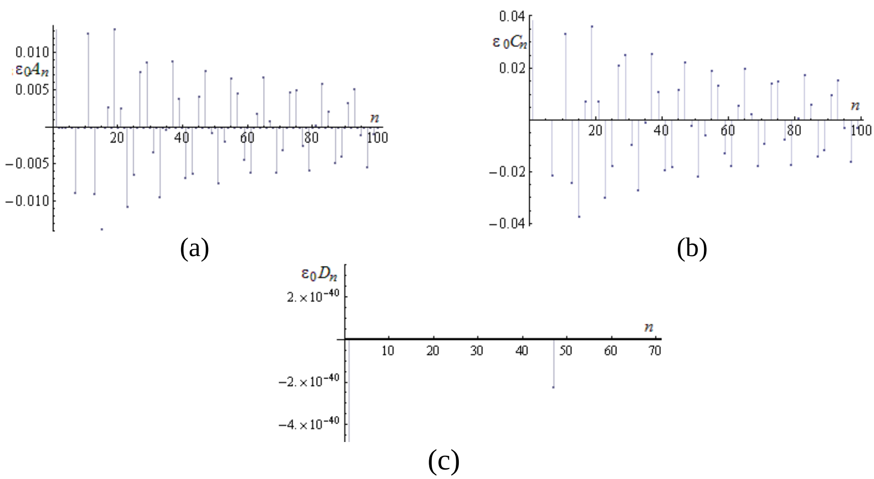
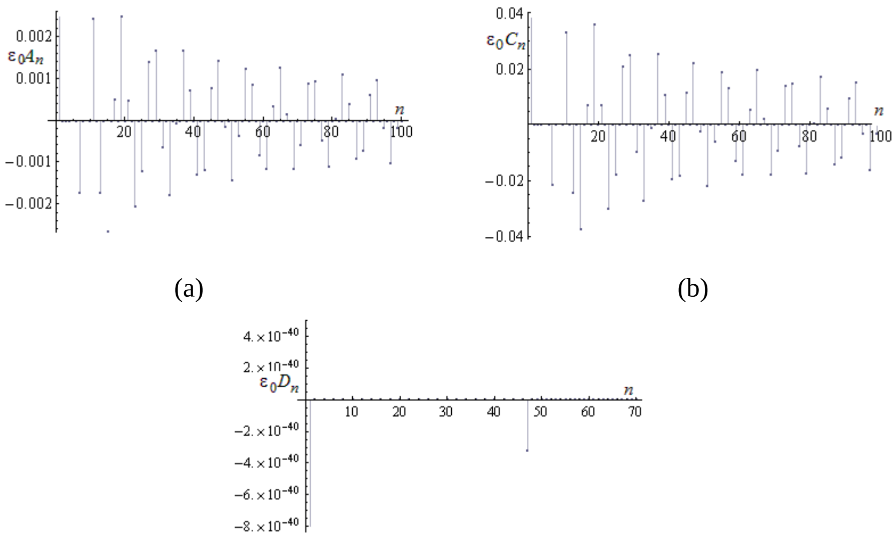


Fig. 2 – *Coefficients* $A_n(n)$, $C_n(n)$ *and* $D_n(n)$, n=1, … , 100, *in expressions* (8) – (10) *for* $b/R=0.4$ *and* ε_r = 2.



(c)

Fig. 3 – *Coefficients* $A_n(n)$, $C_n(n)$ *and* $D_n(n)$, n=1, … , 100,
in expressions (8) – (10) *for* $b/R = 0.4$ *and* $\varepsilon_r = 15$.

Coefficients $A_n(n)$ decrease significantly with increment of relative permittivity of a medium (Figs. 2a and 3a).

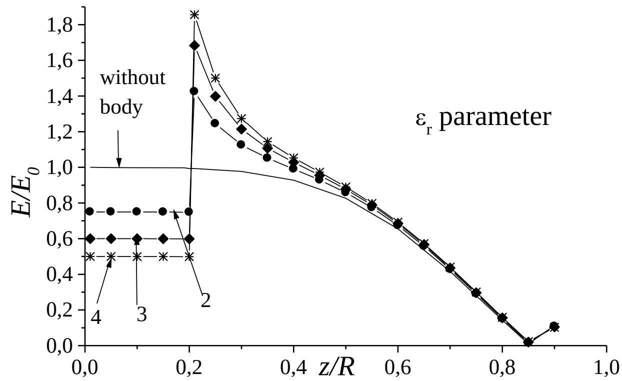
3.2 Influence of a relative permittivity of a dielectric sphere on the electric field homogeneity

In this section we investigate the influence of a relative permittivity of the sphere on the homogeneity of the generated electric field. The influence is assessed by considering a spatial distribution of an electric field.

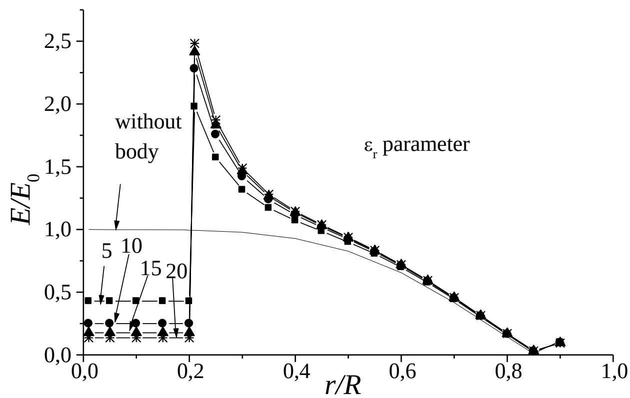
We consider a system, presented in Fig. 1, with b/R = 0.2 and several values of relative permittivity, ε_r = 2, 3, 4, 5, 10, 15, 20. Distribution of the electric field intensity E along the z-axis, normalized by the electric field E_0 at the point $r = 0$ for the system without a sphere and in the case when a dielectric sphere is placed in the center of the system, for ε_r = 2, 3, 4 and for ε_r = 5, 10, 15, 20 is presented in Figs. 4a and 4b, respectively.

Increase of relative permittivity of the sphere, leads to a decrement of the electric field in its interior, Figs. 4a and 4b. By very high permittivity values, the electric field tends to zero. This is in accordance with simplified consideration of medium of very high permittivity as a metal, in which we have the absence of the electric field.

Larger permittivity value of a sphere disrupts more the electric field outside the sphere. This applies up to $z/R = 0.4$ (Figs. 4a and 4b), when the electric field starts to be smaller than the field at the center of the sphere. From that point the ratio E/E_0 is practically the same, regardless of the permittivity value.



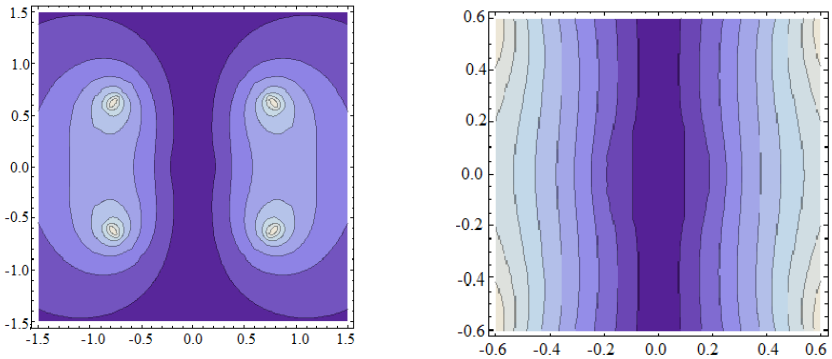
(a) $b/R = 0.2$, $\varepsilon_r = 2, 3, 4$.



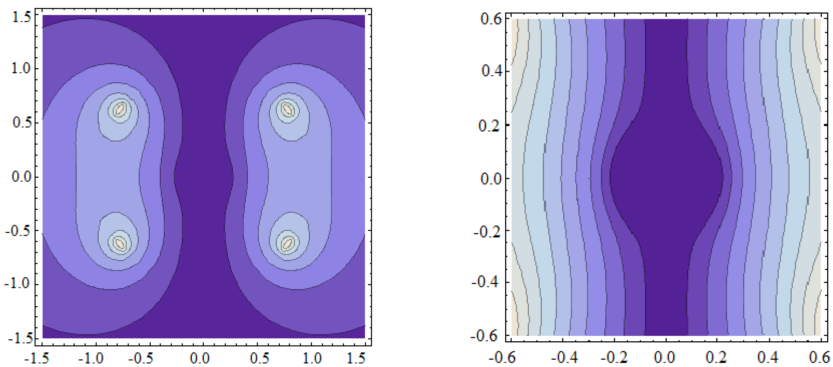
(b) $b/R = 0.2$, $\varepsilon_r = 5, 10, 15, 20$.

Fig. 4 – *Distribution of the electric field strength normalized by the field E_0 at the point $r = 0$ along the z – axis, with a sphere ($b/R = 0.2$) and without a sphere:* (a) $\varepsilon_r = 2, 3, 4$; (b) $\varepsilon_r = 5, 10, 15, 20$.

The same effect can be observed at the shape of equipotential lines: increment of a permittivity of the sphere results in larger changes in the shape of equipotential lines. Equipotential lines in the system of toroidal electrodes and a sphere of a radius $b/R = 0.2$ and a relative permittivity $\varepsilon_r = 2$ and $\varepsilon_r = 20$ are presented in Figs. 5a and 5b, respectively.



(a) $\varepsilon_r = 2$.

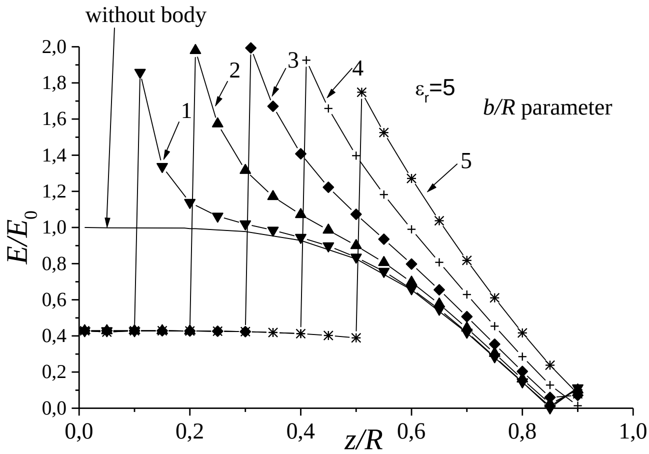


(b) $\varepsilon_r = 20$.

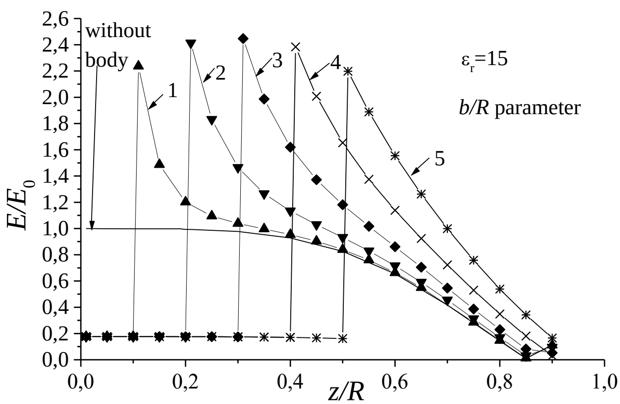
Fig. 5 – *Equipotential lines in the whole system (left) and its central region (right), with a sphere of a radius b/R* = 0.2 *and a relative permittivity* (a) $\varepsilon_r = 2$; (b) $\varepsilon_r = 20$.

3.3 Influence of a radius of a dielectric sphere on the electric field homogeneity

The second study is dedicated to the examination of the influence of a radius of a dielectric sphere on a homogeneity of the electric field produced by the system presented in Fig. 1. We consider different radii of a sphere, $b/R \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$ and two values of its relative permittivity: $\varepsilon_r = 5$ and $\varepsilon_r = 15$. Distribution of the electric field intensity E along the z-axis normalized by the electric field E_0 at the point $r = 0$ for the system without and with a sphere of radius $b/R \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$, for $\varepsilon_r = 5$ and $\varepsilon_r = 15$ is presented in Figs. 6a and 6b, respectively. Ratios $b/R \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$ are in Fig. 6 denoted by 1, 2, 3, 4 and 5, respectively.



(a) $E/E_0(z/R)$, $\varepsilon_r = 5$.

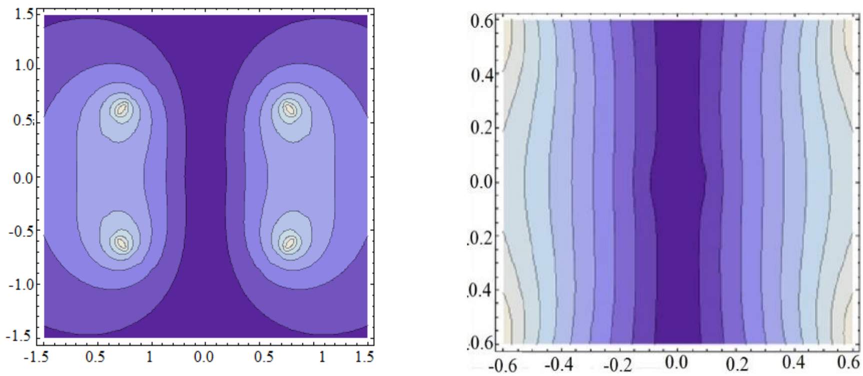


(b) $E/E_0(z/R)$, $\varepsilon_r = 15$

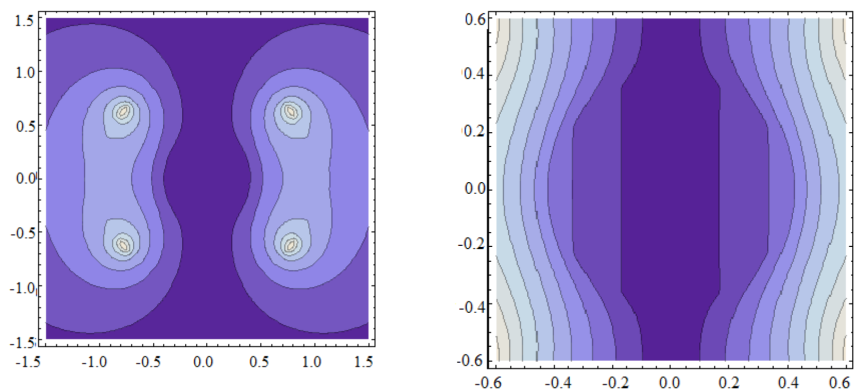
Fig. 6 – *Distribution of the electric field along the* z *– axis normalized by the field* E_0*,* $E/E_0(z/R)$*, for:* (a) ε_r = 5; (b) ε_r = 15.

By observing the Fig. 6, it can be noticed that the increment of a radius of the sphere increases the influence on the homogeneity of electric field in the area outside the largest investigated sphere, $z/R > 0.5$. The same conclusion can be derived for the field distortion at the surface of the sphere itself, when spheres are small, e.g. E/E_0 is larger at the surface of the sphere $b/R = 0.2$ than at the surface of the sphere $b/R = 0.1$. These fields can be compared since the normalized value of the field E/E_0 at points $z/R = 0.1$ and $z/R = 0.2$, when there is no external body, is equal to 1 (black solid line in Fig. 6). Due to increase in deviation of the normalized field E / E_0 going from the center of the system towards the toroidal electrodes, the electric field at the surface of a small sphere, e.g. $b / R = 0.1$, cannot be directly compared to the field distortion at the surface of the sphere of $b / R = 0.5$.

Similar to the distortion of an electric field, a sphere of a larger radius disrupts more the parallel and evenly spaced equipotential lines in the central region between the electrodes. Equipotential lines in the system of toroidal electrodes and a sphere of a relative permittivity $\varepsilon_r = 5$ and a radius $b / R = 0.1$ and $b / R = 0.4$ are presented in Figs. 7a and 7b, respectively.



(a) $b/R = 0.1$



(b) $b/R = 0.4$

Fig. 7 – *Equipotential lines in the whole system (left) and its central region (right), with a sphere of a relative permittivity* $\varepsilon_r = 5$ *and a radius:*
(a) $b/R = 0.1$*; (b)* $b/R = 0.4$.

4 Conclusion

A homogeneous electric field needed by many modern devices and measurement setups is provided by two toroidal electrodes charged by equal charges of opposite signs. This homogeneity can be disturbed by placing of an external body in a field. We model a body by an isotropic dielectric sphere and assess the influence of a relative permittivity and a radius of the sphere on the electric field homogeneity.

Calculation of the electric field in the whole system is done using the method of separation of variables. With the increase of a radius and a permittivity of the sphere, the impact on the homogeneity of the field increases and the field is more disrupted. Only a very small sphere ($b/R < 0.2$) of a small relative permittivity ($\varepsilon_r < 5$) does not significantly change the electric field lines.

The method of separation of variables can handle isotropic, anisotropic and bi-isotropic media.

5 Acknowledgment

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