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Response Analysis on Nonuniform Transmission Line

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Abstract: Transients on a lossless exponential transmission line with a pure resistance load are presented in this paper. The approach is based on the two-port presentation of the transmission line. Using Picard-Carson's method the transmission line equations are solved. The relationship between source voltage and the load voltage in s-domain is derived. All the results are plotted using program package Mathematica 3.0.

Keywords: Exponential transmission line, Transient analysis, Picard-Carson's method, Two-port network.

1 Introduction

The transient analysis in the area of power network elements representable by nonuniform transmission lines, is gaining more importance [1-5]. For the transient analysis of nonuniform lines, the nonuniform transmission line can be treated as a cascading of infinitely short segments of the uniform transmission lines with different characteristic parameters [2, 3]. The second technique is based on extending the concept of the reflection and refraction coefficients creating lattice diagram [3].

If the focus of interests is the propagation of signal or energy on the line the transmission line can be studied as a circuit theory model where voltages and currents are the variables [1], [4] and [5]. The Laplace transform is used for obtaining the closed form of signals and later the s-domain model is transformed into the time domain.

The analysis of transients in transmission line for different voltage sources using circuit theory approach is presented in this paper. The $ABCD$ parameters of transmission line are derived by Picard-Carson's method.

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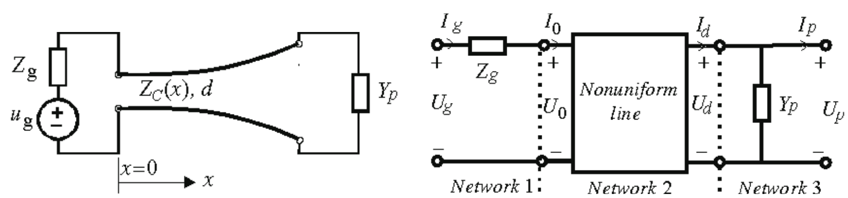
2 Voltage Response

Network Transfer Function

A nonuniform transmission line shown in Fig. 1(a) is considered. The source voltage is U_g and the load voltage is U_p, Z_g and Y_p are source impedance and load admittance, respectively.

Three cascaded two-port networks shown in Fig. 1(b), can represent the system shown in Fig.1 (a). Equivalent $ABCD$ parameters of the system are [4]:

$$\begin{bmatrix} U_g \\ I_g \end{bmatrix} = \begin{bmatrix} CZ_g + DZ_gY_p + BY_p + A & Z_gD + B \\ C + Y_pD & D \end{bmatrix} \begin{bmatrix} U_p \\ I_p \end{bmatrix} \tag{1}$$



(a) (b)

Fig. 1 – *Nonuniform line and its corresponding cascaded two-port networks.*

The relationship between load and source voltage in s-domain can be derived as

$$T_u(s) = \left.\frac{U_p}{U_g}\right|_{I_p = 0} = \frac{1}{A_{ek}} = \frac{1}{CZ_g + DZ_gY_p + BY_p + A}, \tag{2}$$

ABCD Parameters of Distributed Networks

Assuming TEM mode of propagation, the behaviour of the transmission line is described in s-domain by Telegraph's equations:

$$\frac{\mathrm{d}U(s, x)}{\mathrm{d}x} = -Z(x,s)I(s, x) \tag{3}$$

and

$$\frac{\mathrm{d}I(s, x)}{\mathrm{d}x} = -Y(x,s)U(s, x), \tag{4}$$

where $Z(x, s)$ is per-unit length series impedance and $Y(x, s)$ is the per-unit length shunt admittance of the distributed network.

Picard-Carson's method is used to solve transmission line equations (3) and (4). This method is a powerful method in getting a power series solution for the distributed network because it is easy to calculate poles and zeros. This method solves differential equations by an iterative sequence [1]

$$U_n = U_0 - \int_0^x Z(x,s) I_{n-1}(x,s) \mathrm{d}x \tag{5}$$

and

$$I_n = I_0 - \int_0^x Y(x,s) U_{n-1}(x,s) \mathrm{d}x , \tag{6}$$

for $n = 1,2,3,\ldots$, where U_0 and I_0 are the voltage and current at the input port of transmission line, $x = 0$, Fig. 2.

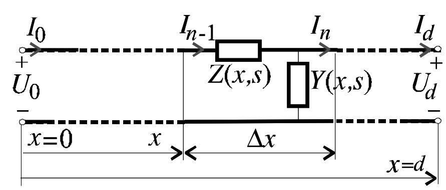


Fig. 2 - *Transmission line and model of elementary line length.*

Since the terms inside the integrals are continuous and bounded, the sequences will converge to the true solutions

$$U(s,x) = \lim_{\Delta x \to 0} U_n(s,x) \tag{7}$$

and

$$I(s,x) = \lim_{\Delta x \to 0} I_n(s,x) . \tag{8}$$

Equations (5) and (6) may be written in the form of

$$U(s,x) = U_0 - \int_0^x Z(x,s) I(s,x) \mathrm{d}x , \tag{9}$$

$$I(s,x) = I_0 - \int_0^x Y(x,s) U(s,x) \mathrm{d}x . \tag{10}$$

Using Picard-Carson's method, these solutions can be presented in the form of two-port parameters [1],

$$U(s,x) = U_0 \sum_{n=0}^{\infty} \zeta_{2n} - I_0 \sum_{n=0}^{\infty} \zeta_{2n+1} \tag{11}$$

$$I(s,x) = U_0 \sum_{n=0}^{\infty} \psi_{2n+1} - I_0 \sum_{n=0}^{\infty} \psi_{2n} \ . \tag{12}$$

From equations (11) and (12), the $A\,B\,C\,D$ parameters are:

$$A(x) = \sum_{n=0}^{\infty} \psi_{2n}, \quad B(x) = \sum_{n=0}^{\infty} \zeta_{2n+1} \ . \tag{13}$$

$$C(x) = \sum_{n=0}^{\infty} \psi_{2n+1} \text{ and } D(x) = \sum_{n=0}^{\infty} \zeta_{2n} \ . \tag{14}$$

$$\zeta_0 = 1 \text{ and } \psi_0 = 1 \tag{15}$$

are values chosen as initial values for iteration starting. The other terms in the summations are found by evaluating the following integrals iteratively:

$$\zeta_n = \int_0^x Z\,\psi_{n-1}\,\mathrm{d}x \text{ and } \psi_n = \int_0^x Y\,\zeta_{n-1}\,\mathrm{d}x \,. \tag{16}$$

Substituting $A\,B\,C\,D$ parameters into (2), the voltage response in s-domain is obtained:

$$U_\mathrm{p}(s) = T_u(s) U_\mathrm{g}(s). \tag{17}$$

The time domain output signals are the inverse Laplace's transforms of the s-domain signals.

3 Numerical Results for the Exponential Transmission Line

A lossless exponential transmission line of known characteristic impedance $Z_\mathrm{C}(x,s) = Z_{\mathrm{C}0}\,\mathrm{e}^{2kx}$, with $k = \frac{1}{2d} \ln M$, where M denotes a taper ratio of exponential transmission line of length d, which is defined as $M = Z_{\mathrm{C}d} / Z_{\mathrm{C}0}$, is observed in this paper. $Z_{\boldsymbol{C}0}$ and $Z_{\mathrm{C}d}$ are the characteristic impedances at the source and load ends, respectively, $Z_{C0} = \sqrt{l_0 / c_0}$. $Z(x,s)$ and $Y(x,s)$ are defined as:

$$Z(x,s) = l_0 s\,\mathrm{e}^{2kx} \text{ and } Y(x,s) = c_0 s\,\mathrm{e}^{-2kx} \,. \tag{18}$$

l_0 is unit length inductance and c_0 is unit length capacitance.

Starting from (15) and using equations (16), a few first terms of series are [4]:

$$\zeta_1(x) = \frac{l_0 s}{2k}\left(e^{2kx} - 1\right), \quad \psi_1(x) = \frac{c_0 s}{2k}\left(e^{-2kx} - 1\right),$$

$$\zeta_2(x) = \frac{l_0 c_0 s^2}{(2k)^2}\left(1 - \left(e^{2kx} - 2kx\right)\right), \quad \psi_2(x) = -\frac{l_0 c_0 s^2}{(2k)^2}\left(1 - \left(e^{-2kx} + 2kx\right)\right),$$

$$\zeta_3(x) = \frac{l_0^2 c_0 s^3}{(2k)^3}\left(2 - 2e^{2kx} + \left(1 + e^{2kx}\right)2kx\right),$$

$$\psi_3(x) = -\frac{l_0 c_0^2 s^3}{(2k)^3}\left(2 - e^{-2kx}\left(2 + \left(1 + e^{2kx}\right)2kx\right)\right),$$

$$\zeta_4 = \frac{l_0^2 c_0^2 s^4}{(2k)^4}\left(3 + \frac{1}{2}\left(-6e^{2kx} + 4\left(2 + e^{2kx}\right)kx\right) + 4k^2x^2\right),$$

$$\psi_4 = \frac{l_0^2 c_0^2 s^4}{(2k)^4}\left(3 + \frac{1}{2}\left(-6e^{-2kx} - 4\left(2 + e^{-2kx}\right)kx\right) + e^{2kx}4k^2x^2\right)$$

$$\vdots$$

When $M = 2$, the numerical results for $ABCD$ parameters are:

$$A(d) = \sum_{n=0}^{\infty} \psi_{2n}(d) = 1 + \psi_2 + \psi_4 + \psi_6 + \cdots \approx$$

$$\approx 1 + 0.40201(s\tau)^2 + 0.03188(s\tau)^4 + 0.001039(s\tau)^6,$$

$$B(d) = \sum_{n=0}^{\infty} \xi_{2n+1}(d) = \zeta_1 + \zeta_3 + \zeta_5 + \cdots \approx$$

$$\approx Z_{C0}\left[1.4427s\tau + 0.23855(s\tau)^3 + 0.011887(s\tau)^5\right],$$

$$C(d) = \sum_{n=0}^{\infty} \psi_{2n+1}(d) = \psi_1 + \psi_3 + \psi_5 + \cdots \approx$$

$$\approx \frac{1}{Z_{C0}}\left[0.721348s\tau + 0.11927(s\tau)^3 + 0.011887(s\tau)^5\right],$$

$$D(d)=\sum_{n=0}^{\infty}\xi_{2n}(d)=1+\zeta_2+\zeta_4+\zeta_6+\cdots\approx$$

$$\approx 1+0.638674\,(s\tau)^2+0.055517\,(s\tau)^4+0.001883\,(s\tau)^6,$$

where $\tau=d\sqrt{l_0 c_0}$ is the time constant.

For the case when $Z_{\mathrm{g}}=50\,\Omega$, $Z_{\mathrm{p}}=100\,\Omega$ and

$$u_{\mathrm{g}}(t)=h(t)\,, \tag{19}$$

the unit step voltage response is presented in Fig. 3. The steady-state value of the load voltage is, expectedly, equal to the final value that at the source end, which is $1/(1+Z_{\mathrm{g}}Y_{\mathrm{p}})\approx 0.6667$, [2, 3].

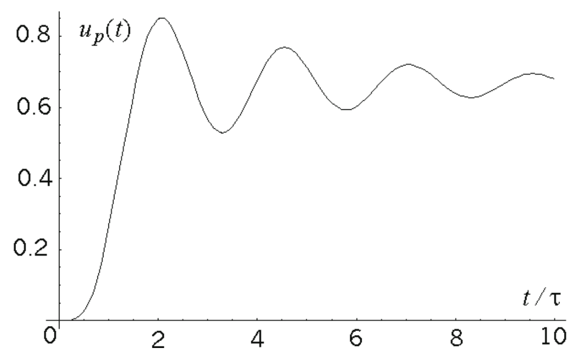


Fig. 3 – *Unit step voltage response.*

When the source voltage is of exponential form

$$u_{\mathrm{g}}(t)=\mathrm{e}^{-at}\,h(t), \tag{20}$$

time dependence of voltage response for $a=5\,\tau$, is shown in Fig. 4.

When the source is rectangular signal

$$u_{\mathrm{g}}(t)=h(t)-h(t-a), \tag{21}$$

of unit amplitude and duration $a=10\,\tau$, transient response output voltage is presented in Fig. 5.

When the source voltage is

$$u_{\mathrm{g}}(t)=t[h(t)-h(t-a)], \tag{22}$$

time dependence of voltage response for $a=5\,\tau$, is shown in Fig. 6.

In the case of source voltage

$$u_g(t) = th(t) - (t-a)h(t-a), \tag{23}$$

time dependence of voltage response for $a = 5\tau$, is shown in Fig. 7.

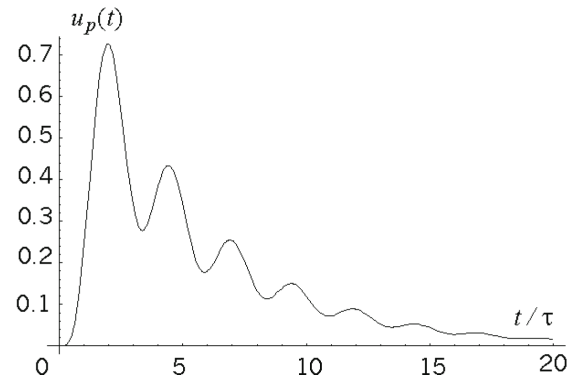


Fig. 4 – *Exponential transient voltage response.*

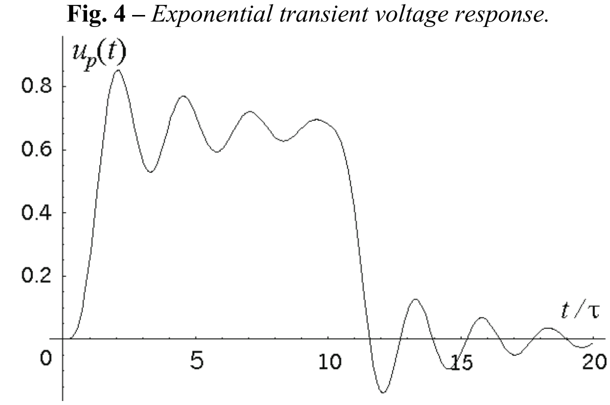


Fig. 5 – *Voltage response on the signal (21).*

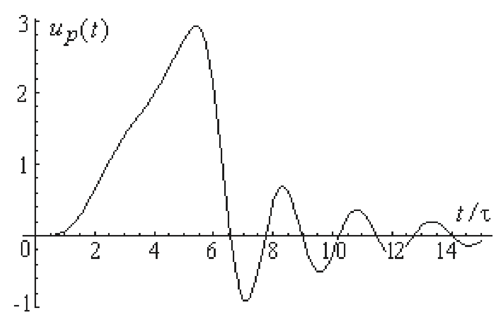


Fig. 6 – *Voltage response on the signal (22).*

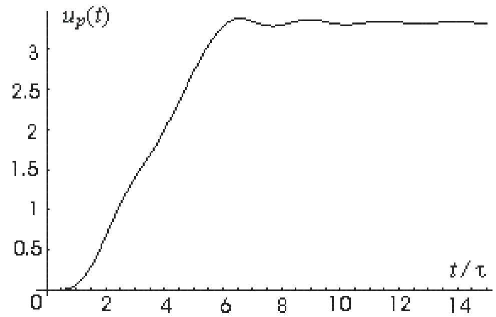


Fig. 7 – *Voltage response on the signal (23).*

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